Alfvén Instabilities Driven by Energetic Particles in Toroidal Magnetic Fusion

Satellite measurements of Alfvén waves that propagate from the magnetosphere to the ionosphere

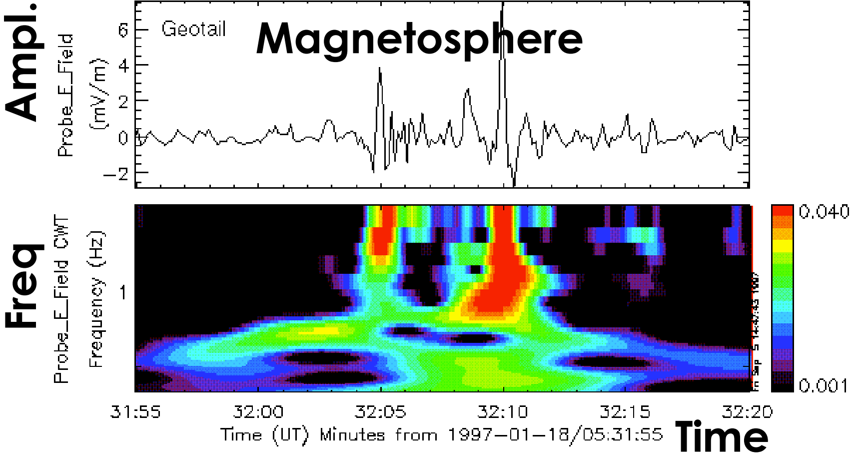


Figure 3 (a)

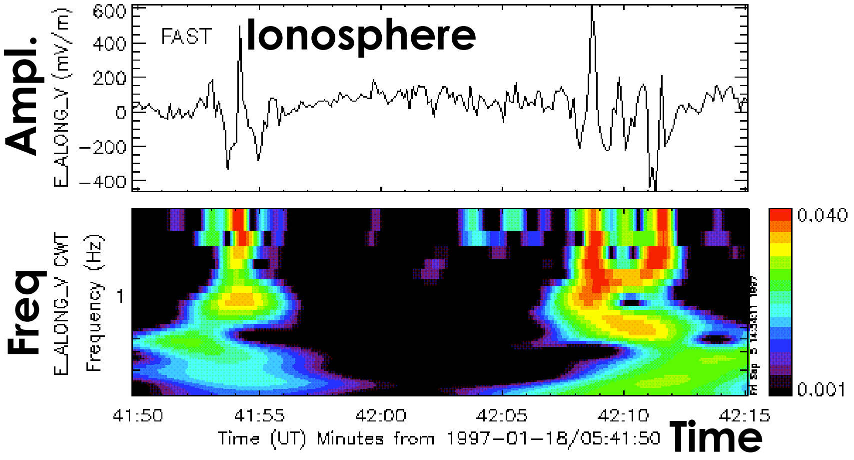
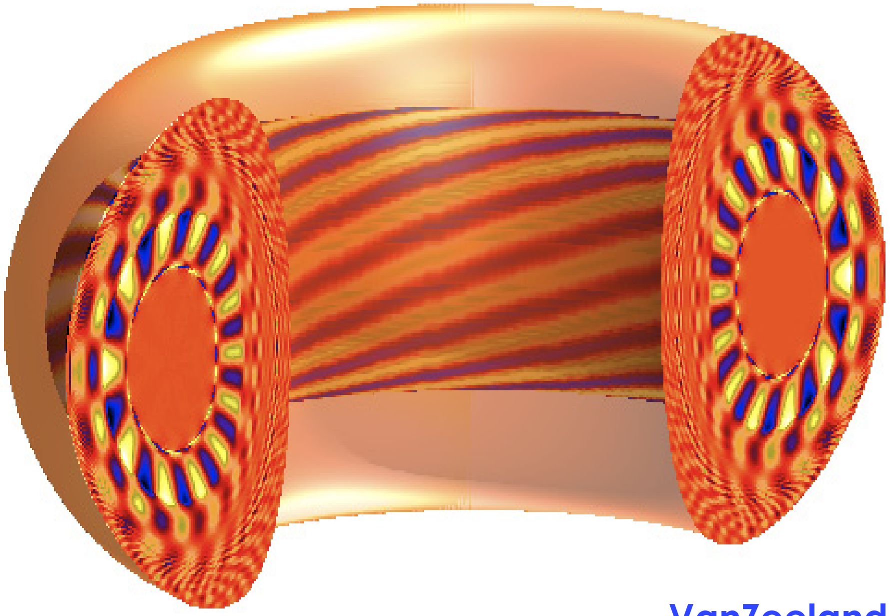


Figure 3 (b)

Sigsbee, Geophys. Res. Lett. 25 (1998) 2077

APS-DPP 11/07

Calculated Alfvén Eigenmode structure in ITER



VanZeeland

W.W. (Bill) Heidbrink

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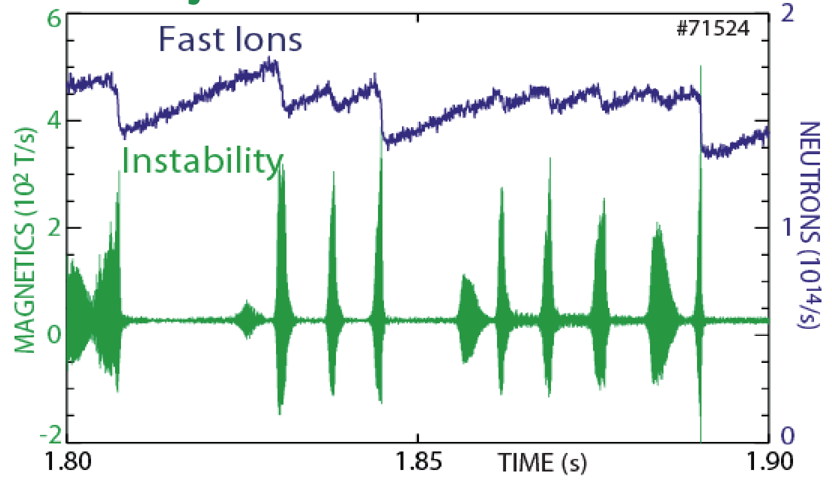
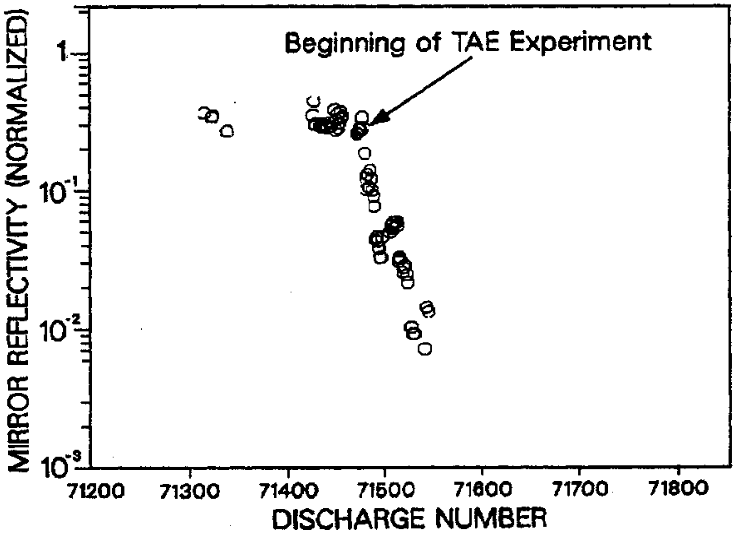
University of California, Irvine

Instabilities Driven by Energetic Particles are of both scientific and practical interest

Beam injection into the DIII-D tokamak





Damage

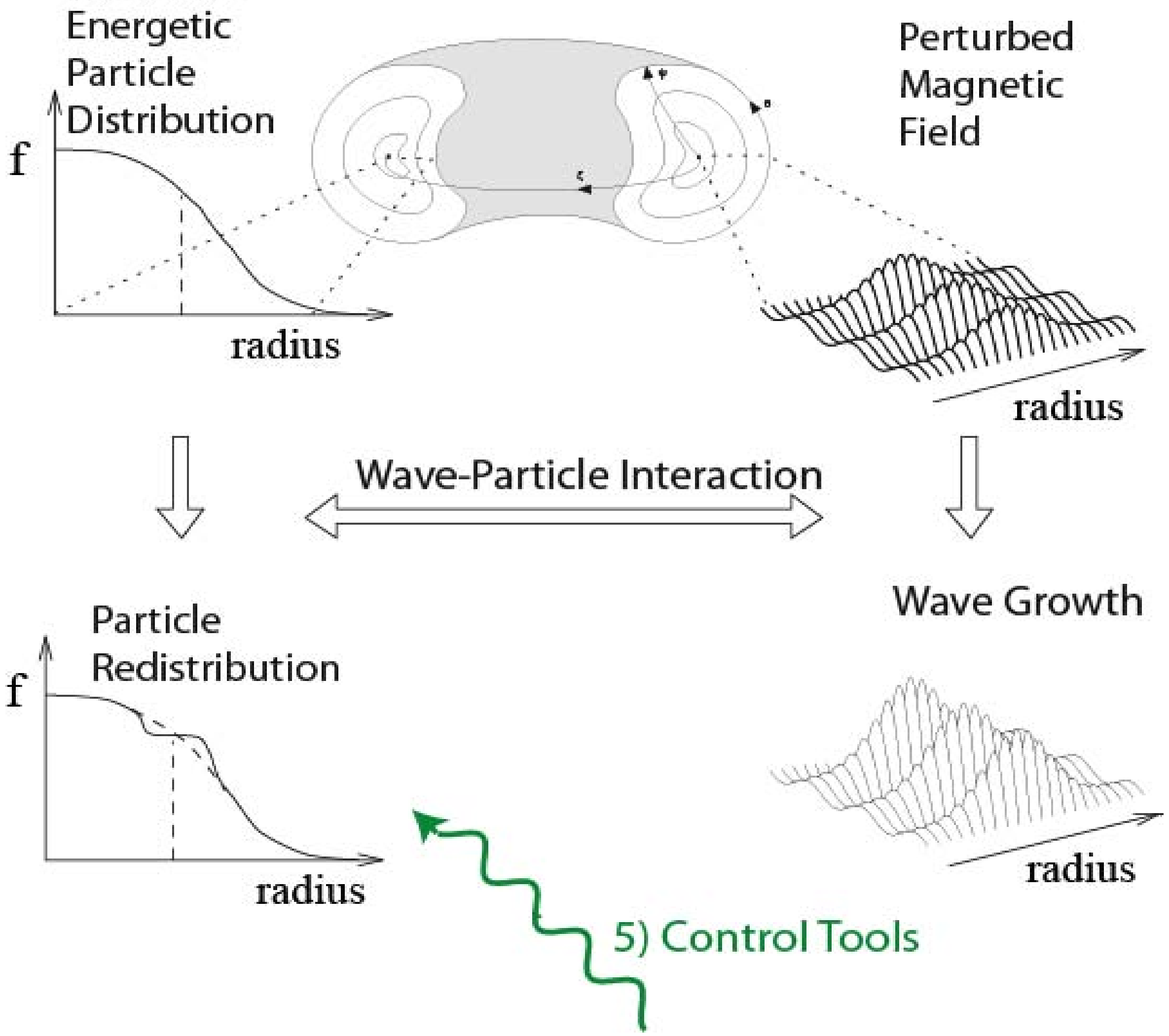
- Carbon coats DIII-D mirrors when escaping fast ions ablate the graphite wall[1]
- Transport of fast ions by Alfvén waves onto unconfined orbits cause a vacuum leak in TFTR[2]

Losses of charged fusion products must be controlled in a reactor!

[1] Duong, Nucl. Fusion 33 (1993) 749.

[2] White, Phys. Pl. 2 (1995) 2871.

Outline



Energetic Particle Distribution

Perturbed Magnetic Field

Wave-Particle Interaction

Particle Redistribution

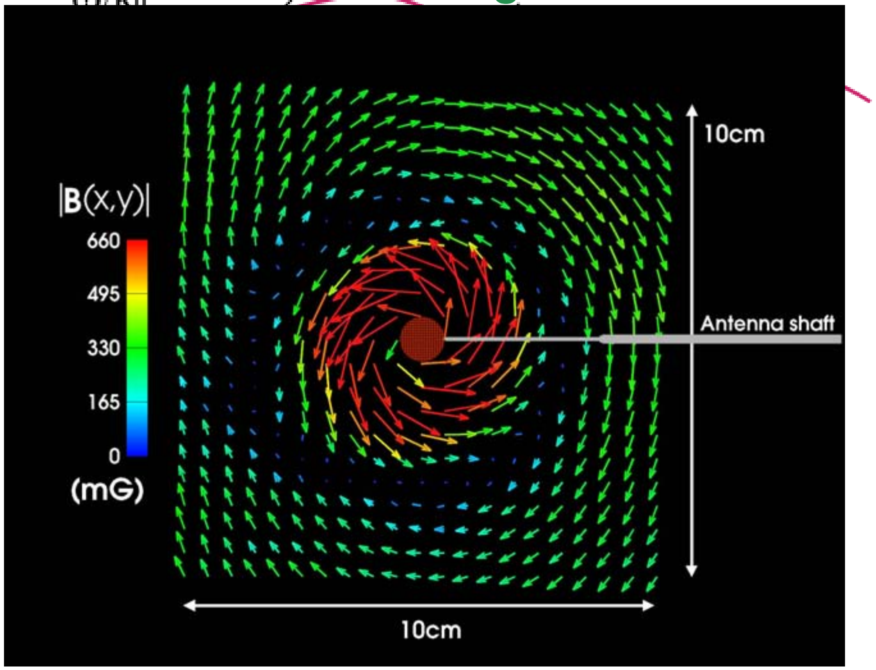
Wave Growth

5) Control Tools

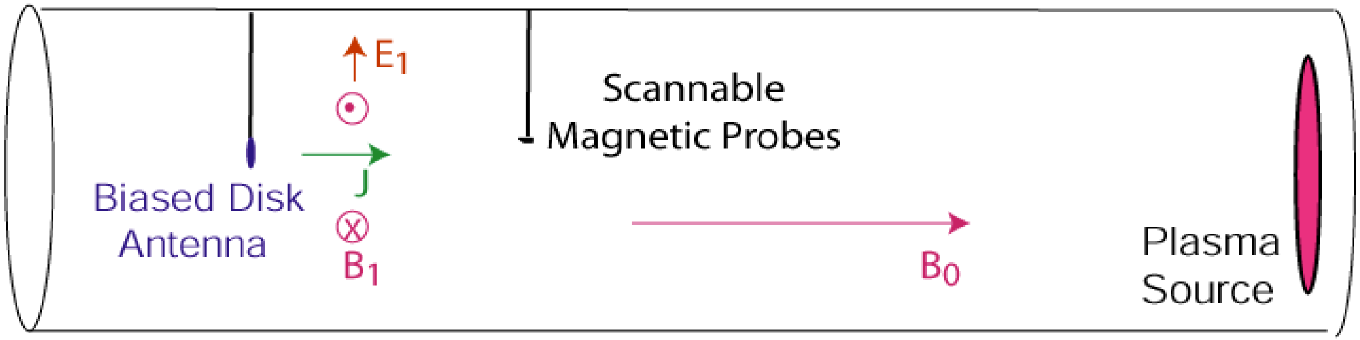
1. Alfvén Gap Modes
2. **Energetic Particles (EP)**
3. **Energetic Particle Modes (EPM)**
4. **Nonlinear Dynamics**
5. **Prospects for Control**

Shear Alfvén Waves are transverse electromagnetic waves that propagate along the magnetic field

Measured circularly polarized shear Alfvén wave in the Large Plasma Device



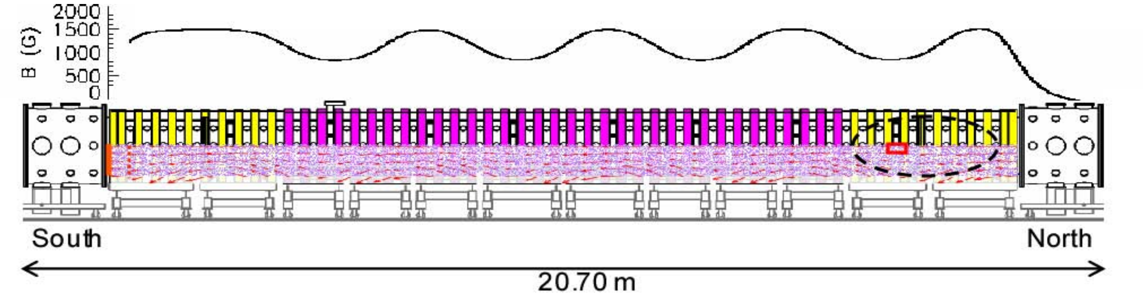
- Dispersionless: $\omega = k_{\parallel} v_A$
- Alfvén speed: $v_A = B/(\mu_0 n_i m_i)^{1/2}$
- $E_{\parallel}$ tiny for $\omega << \Omega_i$
- Particles move with field line
- Analogous to waves on string with B^2 the tension and the mass density provided by the plasma
- All frequencies below Ω_i propagate



Gekelman, Plasma Phys. Cont. Fusion 39 (1997) A101

Periodic variation of the magnetic field produces periodic variations in N for shear Alfvén waves

Periodic Mirror Field in the LAPD

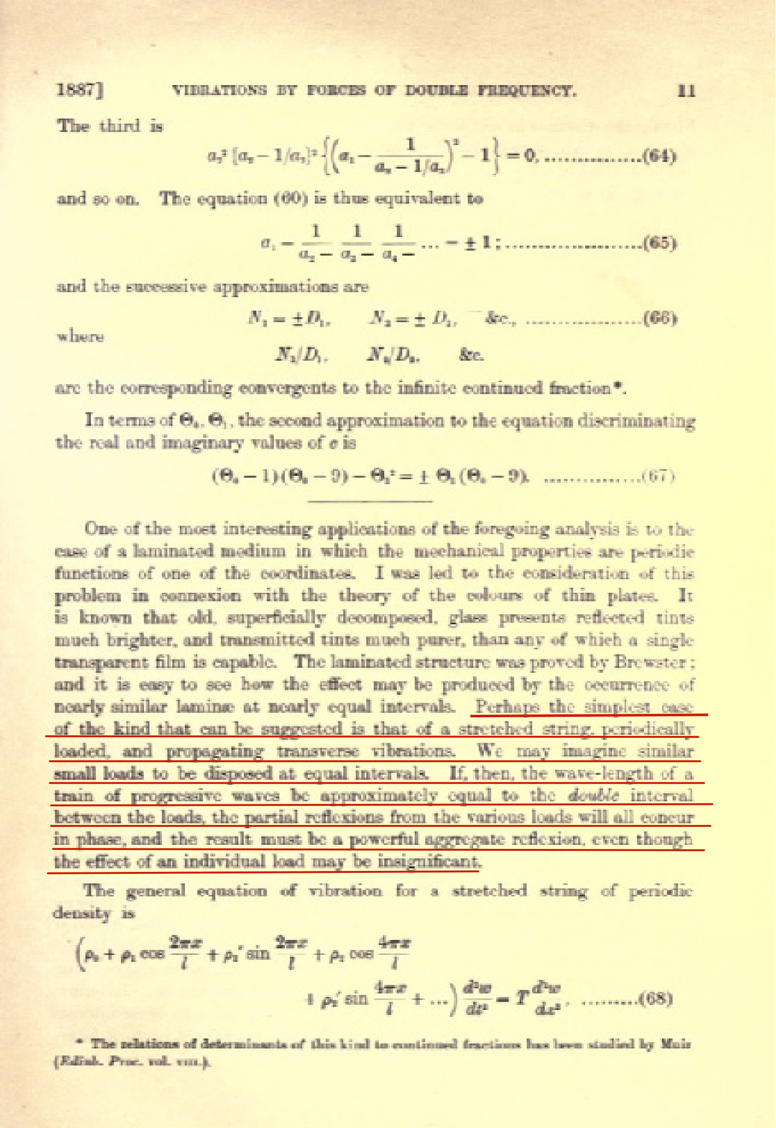


Zhang, Phys. Plasmas 14 (2007)

Periodic variation in B →
Periodic variation in v_A →
Periodic variation in index of refraction N

→ Frequency gap that is proportional to ΔN

Periodic index of refraction → a frequency gap



1887] VIBRATIONS BY FORCES OF DOUBLE FREQUENCY. 11

The third is

$$a_3^2\,[a_2-1/a_3]^2\left\{\left(a_1-\frac{1}{a_2-1/a_3}\right)^2-1\right\}=0, \quad\text{.................(64)}$$

and so on. The equation (60) is thus equivalent to

$$a_1-\frac{1}{a_2-}\,\frac{1}{a_3-}\,\frac{1}{a_4-}\cdots=\pm 1; \quad\text{........................(65)}$$

and the successive approximations are

$$N_1=\pm D_1, \qquad N_2=\pm D_2, \qquad \&c., \quad\text{...................(66)}$$

where

$$N_1/D_1, \qquad N_2/D_2, \qquad \&c.$$

are the corresponding convergents to the infinite continued fraction*.

In terms of Θ_0, Θ_1, the second approximation to the equation discriminating the real and imaginary values of e is

$$(\Theta_0-1)(\Theta_0-9)-\Theta_1^2=\pm\,\Theta_1(\Theta_0-9). \quad\text{...............(67)}$$

One of the most interesting applications of the foregoing analysis is to the case of a laminated medium in which the mechanical properties are periodic functions of one of the coordinates. I was led to the consideration of this problem in connexion with the theory of the colours of thin plates. It is known that old, superficially decomposed, glass presents reflected tints much brighter, and transmitted tints much purer, than any of which a single transparent film is capable. The laminated structure was proved by Brewster; and it is easy to see how the effect may be produced by the occurrence of nearly similar laminæ at nearly equal intervals. Perhaps the simplest case of the kind that can be suggested is that of a stretched string, periodically loaded, and propagating transverse vibrations. We may imagine similar small loads to be disposed at equal intervals. If, then, the wave-length of a train of progressive waves be approximately equal to the *double* interval between the loads, the partial reflexions from the various loads will all concur in phase, and the result must be a powerful aggregate reflexion, even though the effect of an individual load may be insignificant.

The general equation of vibration for a stretched string of periodic density is

$$\left(\rho_0+\rho_1\cos\frac{2\pi x}{l}+\rho_1'\sin\frac{2\pi x}{l}+\rho_2\cos\frac{4\pi x}{l}+\rho_2'\sin\frac{4\pi x}{l}+\dots\right)\frac{d^2w}{dt^2}=T\frac{d^2w}{dx^2}, \quad\text{.........(68)}$$

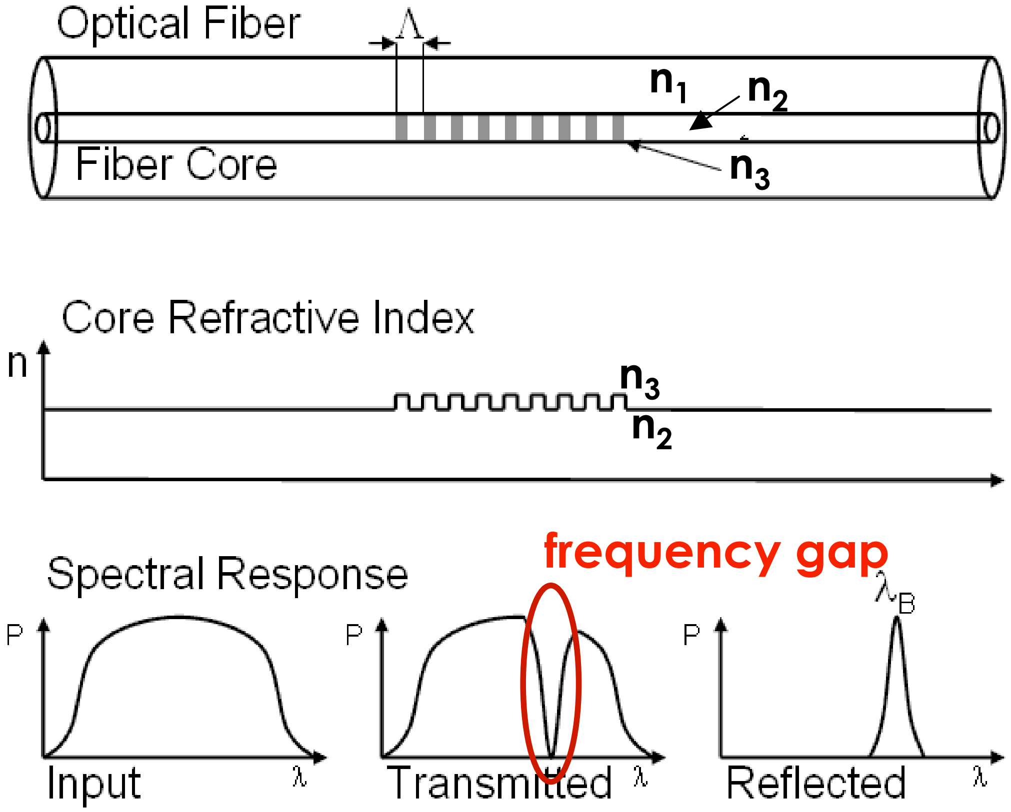
---

* The relations of determinants of this kind to continued fractions has been studied by Muir (*Edinb. Proc.* vol. VIII.).

Lord Rayleigh, Phil. Mag. (1887)

"Perhaps the simplest case … is that of a stretched string, periodically loaded, and propagating transverse vibrations. …If, then, the wavelength of a train of progressive waves be approximately equal to the *double* interval between the beads, the partial reflexions from the various loads will all concur in phase, and the result must be a powerful aggregate reflexion, even though the effect of an individual load may be insignificant."

The propagation gap occurs at the Bragg frequency & its width is proportional to ΔN



frequency gap

Wikipedia, “Fiber Bragg grating”

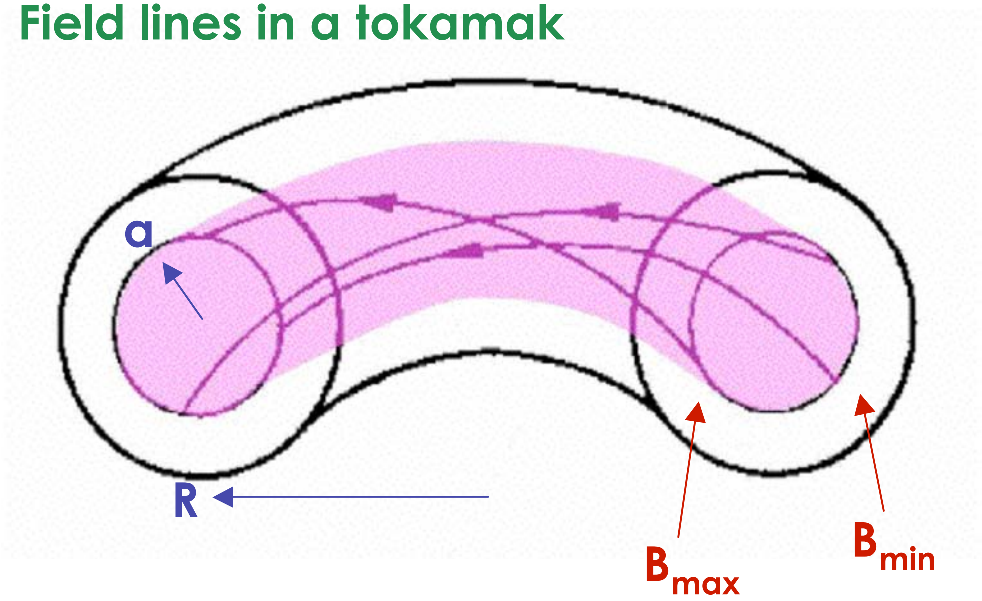
- Destructive interference between counter propagating waves
- Bragg frequency: $f=v/2\Lambda$
- $\Delta f/f \sim \Delta N/N$

for shear Alfvén waves

- $f = v_A/\ 2\Lambda$, where Λ is the distance between field maxima along the field line
- $\Delta f \sim \Delta B/B$

Frequency gaps are unavoidable in a toroidal confinement device

Field lines in a tokamak



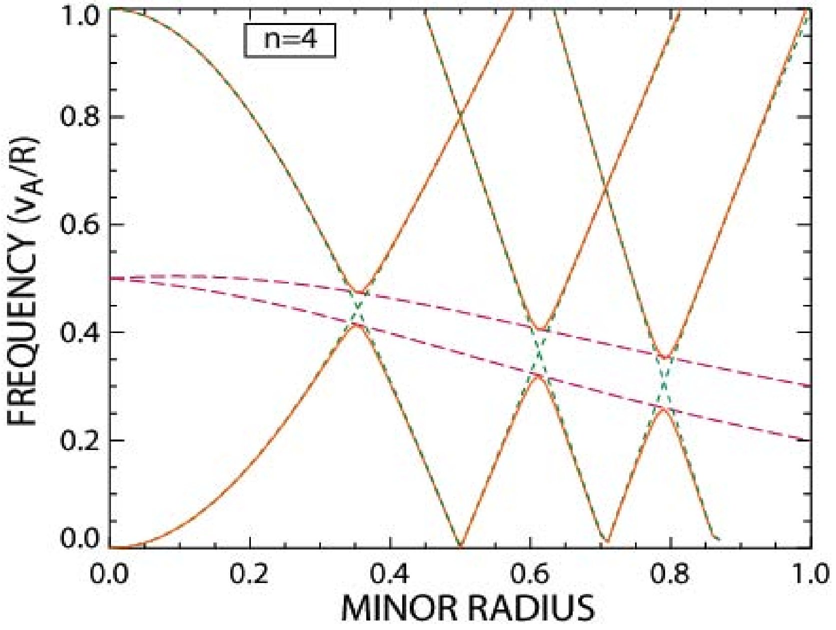
- For single-particle confinement, field lines rotate.
- *Definition: One poloidal transit occurs for every q toroidal transits (q is the "safety factor")*
- B ~ 1/R
- ΔB ~ a/R
- Distance between maxima is $\Lambda = q\,(2\pi R)$ so $f_{gap} = v_A/4\pi qR$

Periodicity constraint on the wavevector: ~ $e^{i\,(n\zeta - m\theta)}$

- n "toroidal mode number"
- m "poloidal mode number"
- ζ and θ toroidal and poloidal angles

Frequency Gaps and the Alfvén Continuum depend on position

Gap caused by toroidicity[1]



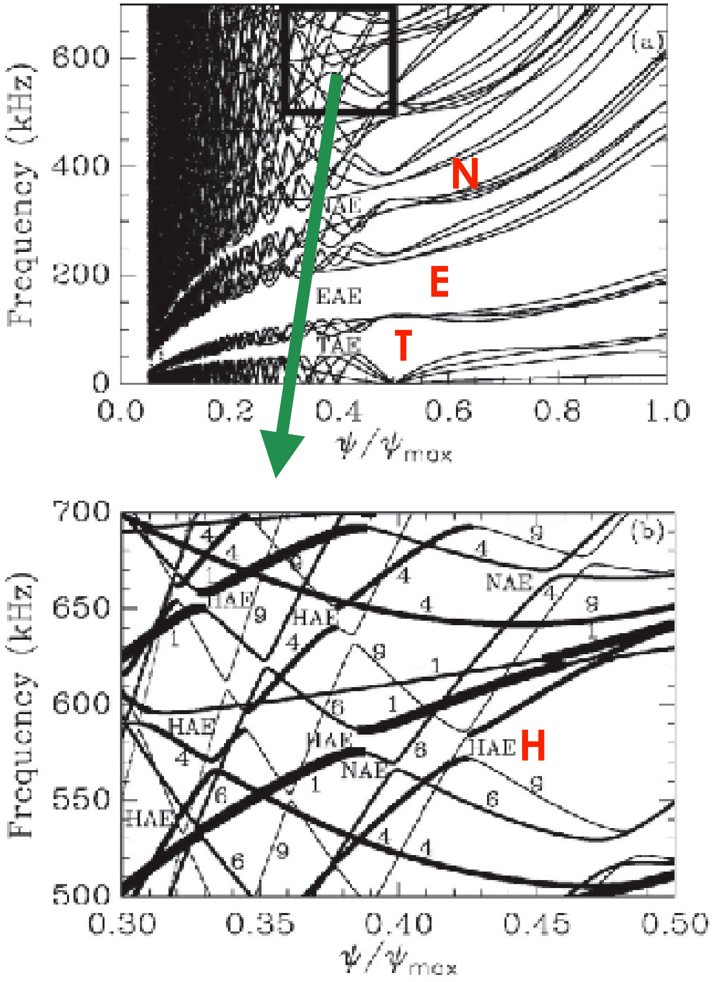
[1] based on Fu & VanDam, Phys. Fl. B1 (1989) 1949

- Centered at Bragg frequency v_A/qR
- Function of position through v_A & q
- Gap proportional to r/R
- If no toroidicity, continuum waves would satisfy $\omega = k_{||} v_A$ with $k_{||} \sim |n - m/q|$
- Counter-propagating waves cause frequency gap
- Coupling avoids frequency crossing (waves mix)
- Crossings occur at many positions

All periodic variations introduce frequency gaps

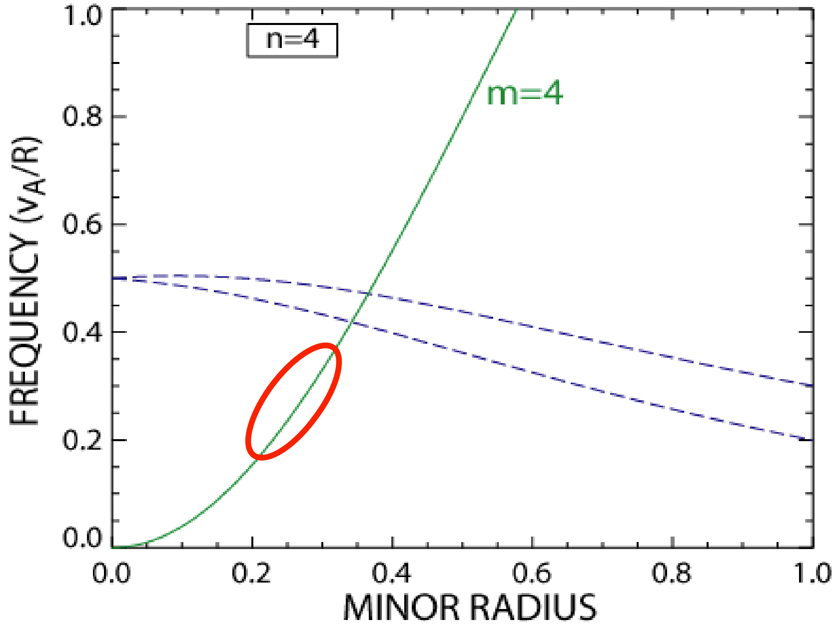
BAE	“beta”	compression
TAE	“toroidicity”	m & m+1
EAE	“ellipticity”	m & m+2
NAE	“noncircular”	m & m+3
MAE	“mirror”	n & n+1
HAE	“helicity”	both n’s & m’s

Shear Alfvén wave continuua in an actual stellarator



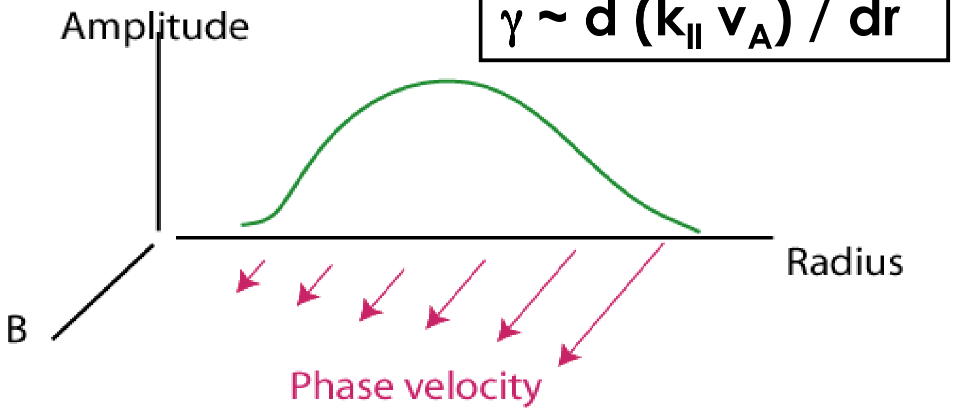
Spong, Phys. Plasmas 10 (2003) 3217

Rapid dispersion strongly damps waves in the continuum



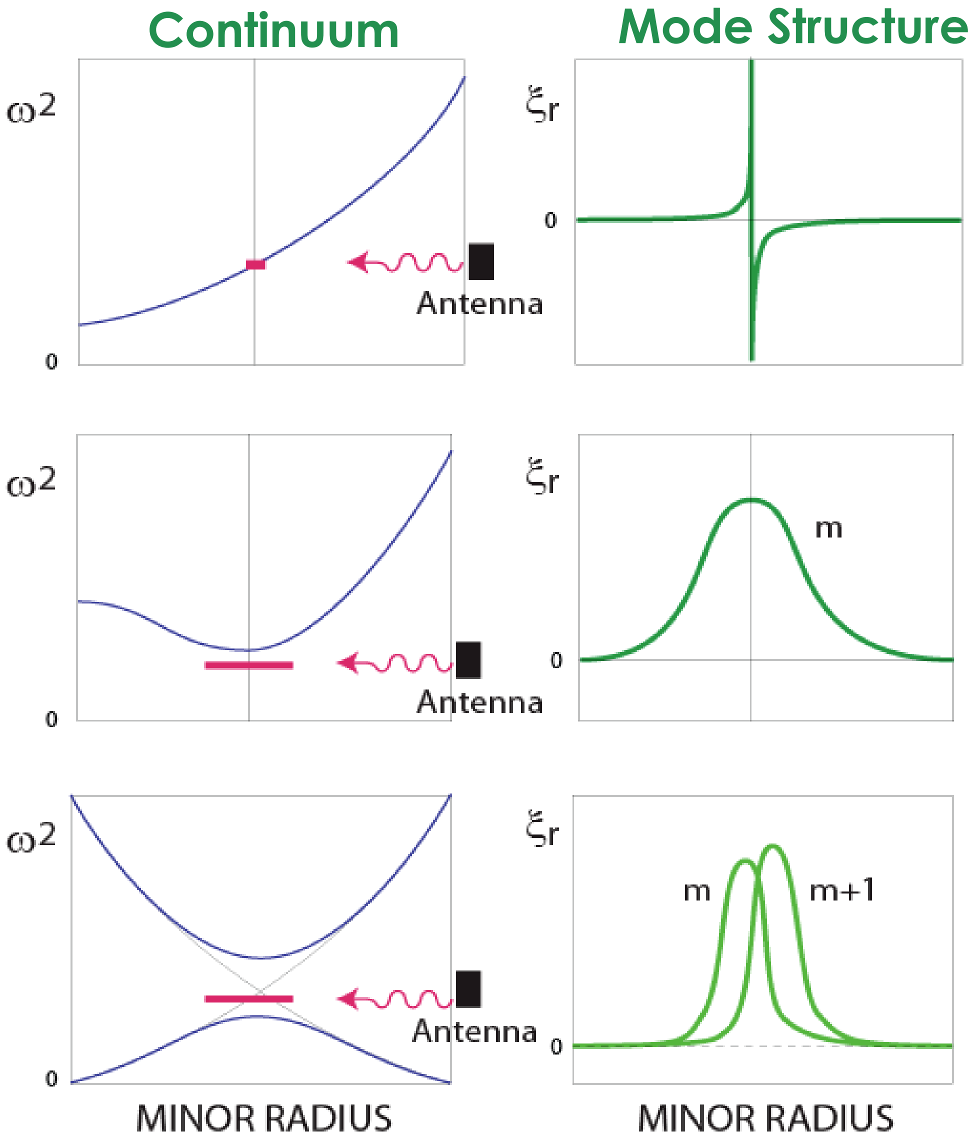
Radially extended modes in the continuum gaps are more easily excited

$$\gamma \sim d\,(k_{\parallel}\, v_A)\,/\,dr$$



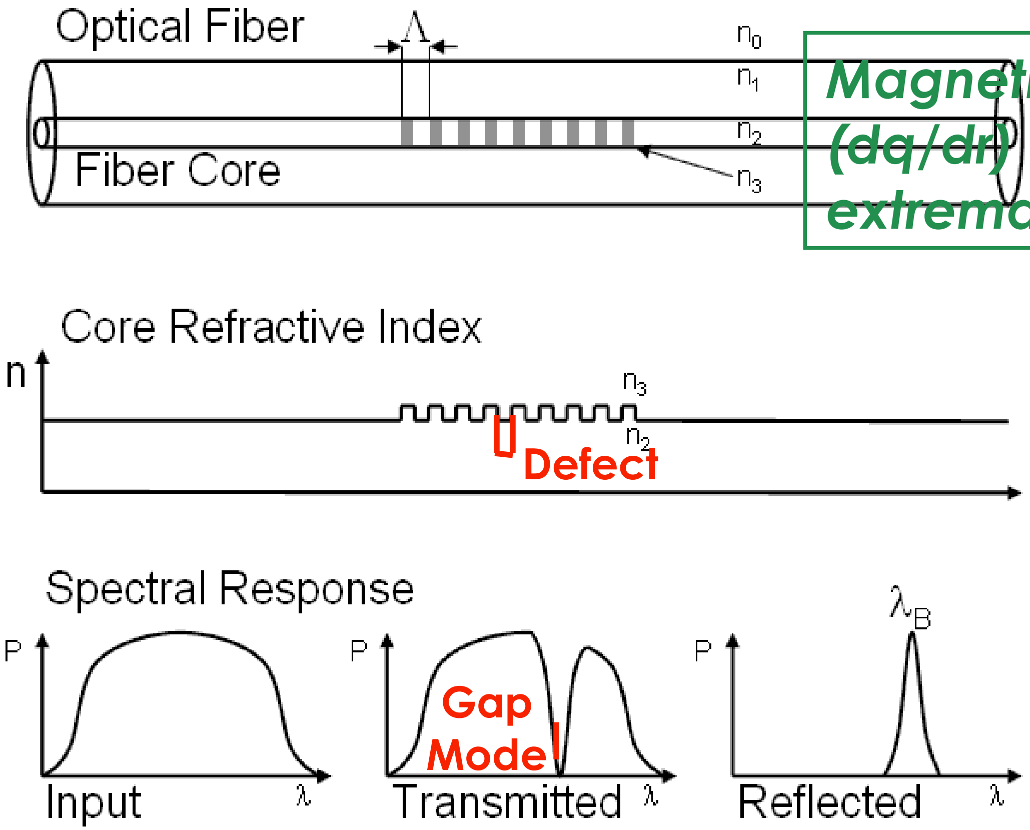
Radially extended Alfvén eigenmodes are more easily excited

- Imagine exciting a wave with an antenna--how does the system respond?
- In continuum, get singular mode structure that is highly damped (small amplitude)
- Where gap modes exist, the eigenfunction is regular & spatially extended



Pinches, Ph.D. Thesis

Magnetic shear is the “defect” that creates a potential well for Alfvén gap modes



Magnetic shear (dq/dr) creates extrema

•In photonic crystals
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mo

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an e
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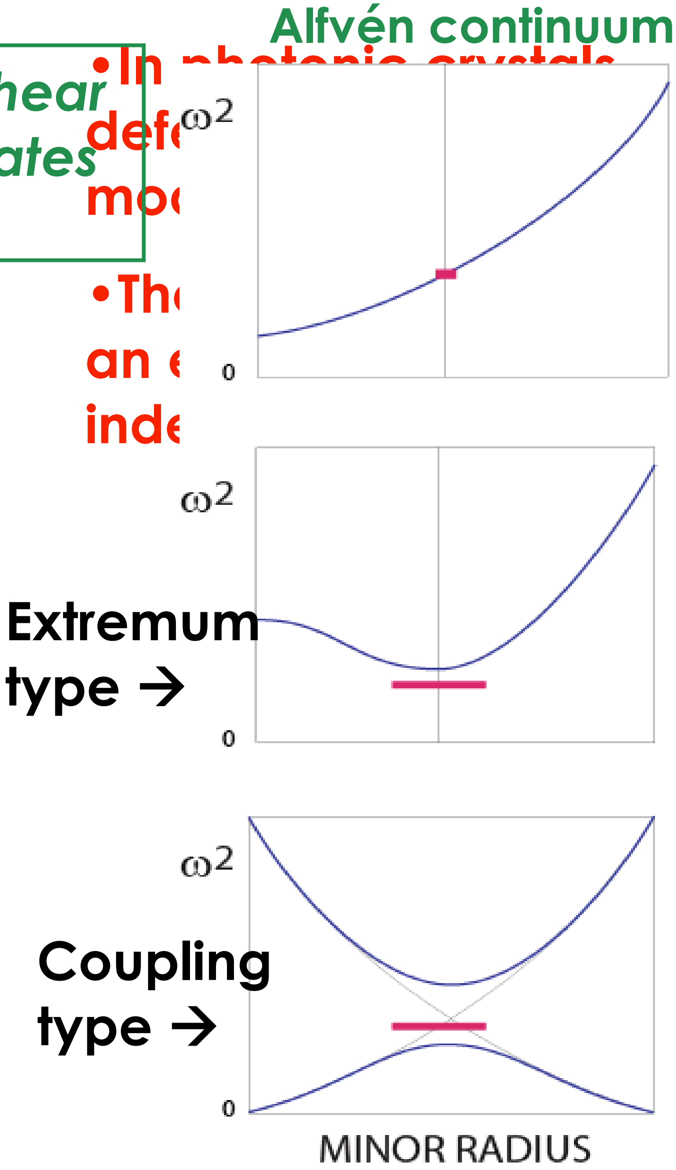
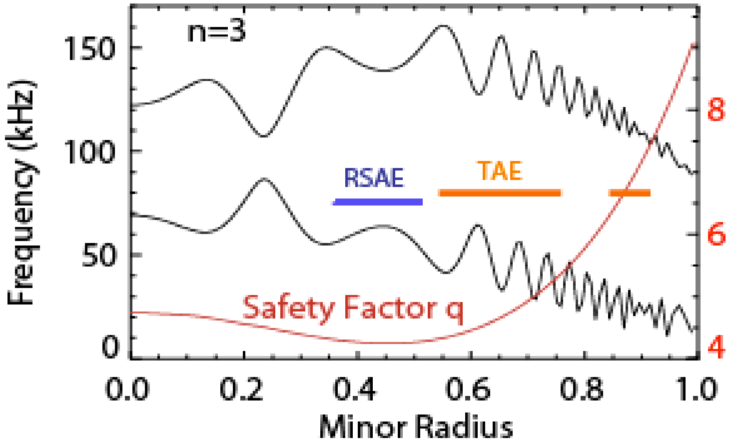


An extremum in the continuum can be the "defect"

Gap structure in a DIII-D plasma with a minimum in the q profile

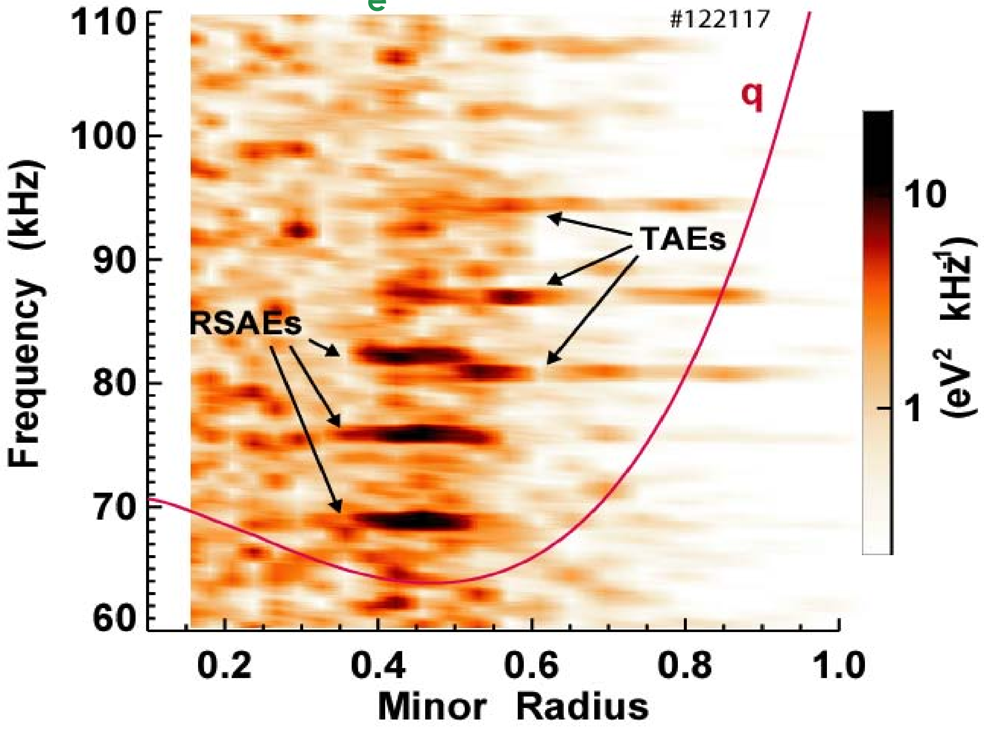


VanZeeland, PRL 97 (2006) 135001; Phys. Plasmas 14 (2007) 156102.

- Many RSAEs with different n's
- All near minimum of measured q
- Structure agrees quantitatively with MHD calculation

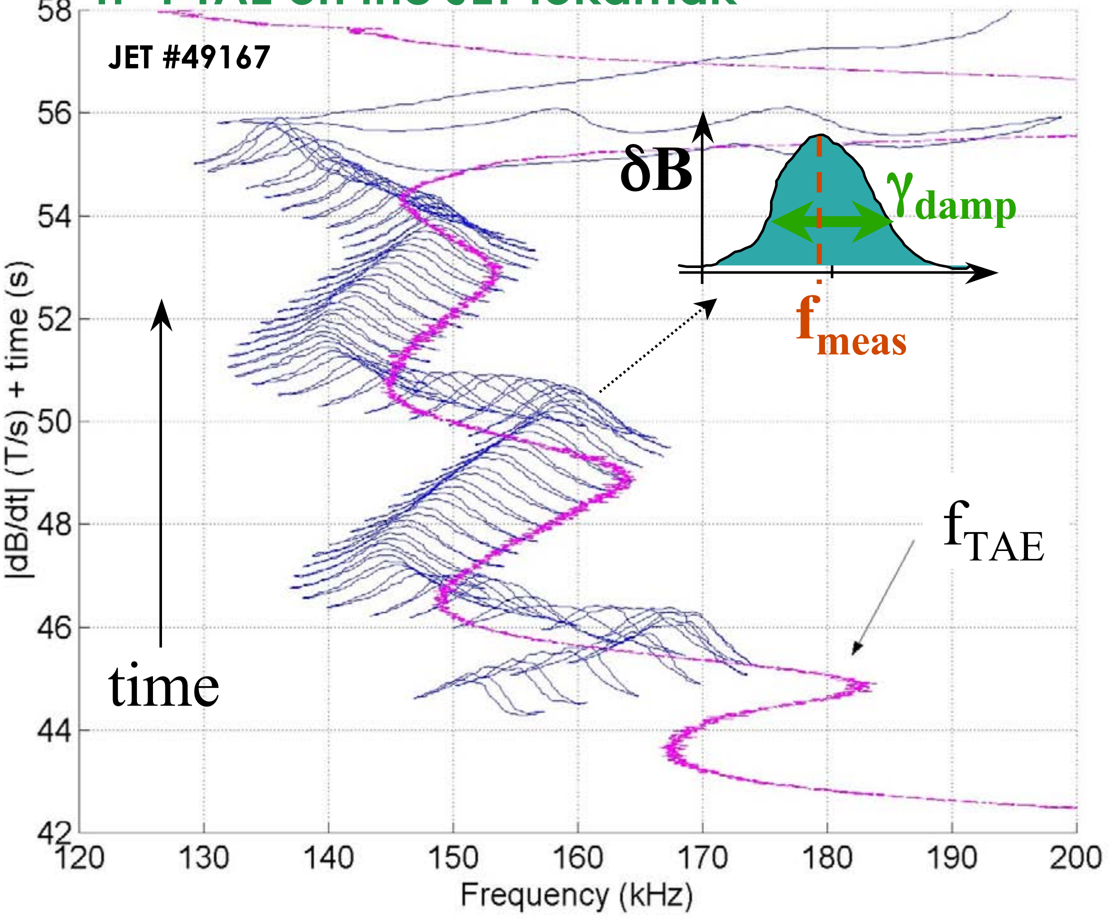
- Gap modes reside in effective waveguide caused by minimum in q profile
- These gap modes called "Reversed Shear Alfvén Eigenmodes" (RSAE)

Measured δT_e mode structure in DIII-D



In the toroidal Alfvén Eigenmode (TAE), mode coupling is the “defect” that localizes the mode

Using an external antenna to excite a n=1 TAE on the JET tokamak

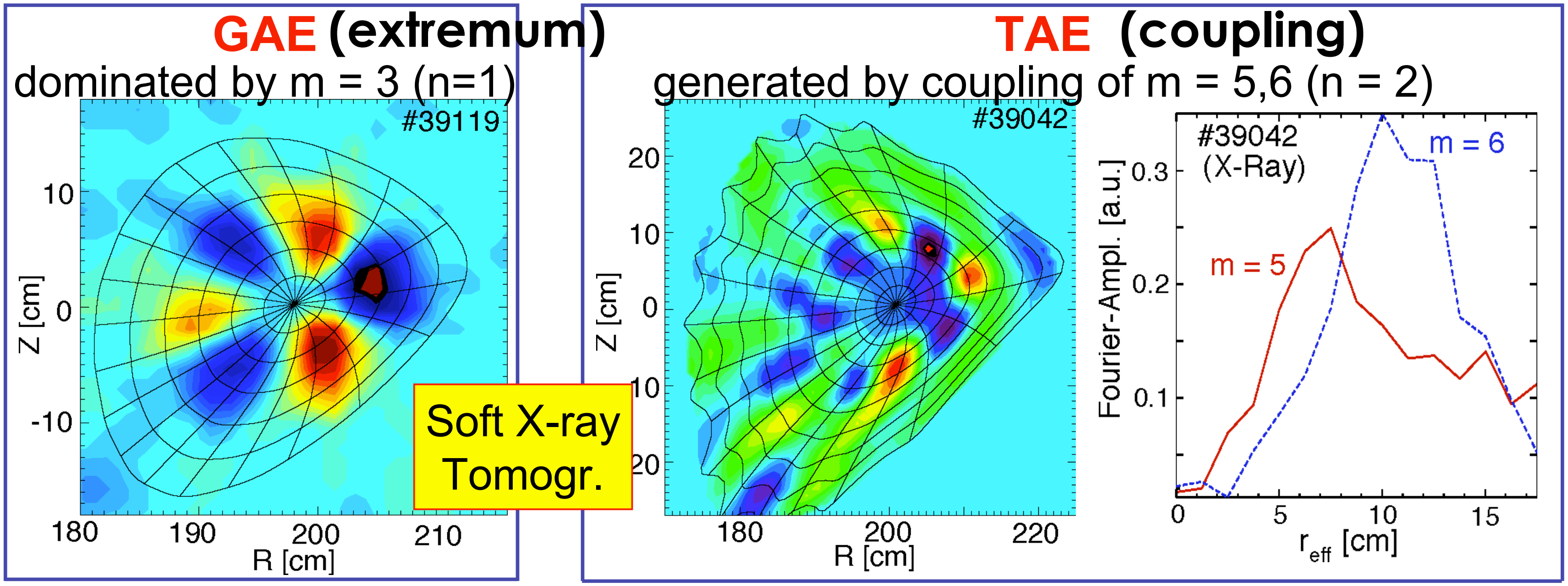


- The frequency of the measured TAE follows the frequency gap as the discharge evolves
- Can infer the wave damping from the width of the resonance
- Width is larger when the eigenfunction “touches” the continuum

Fasoli, Phys. Plasmas 7 (2000) 1816

Predicted spatial structure is observed experimentally for both types of gap mode

Data from W7-AS stellarator


GAE (extremum)

dominated by m = 3 (n=1)

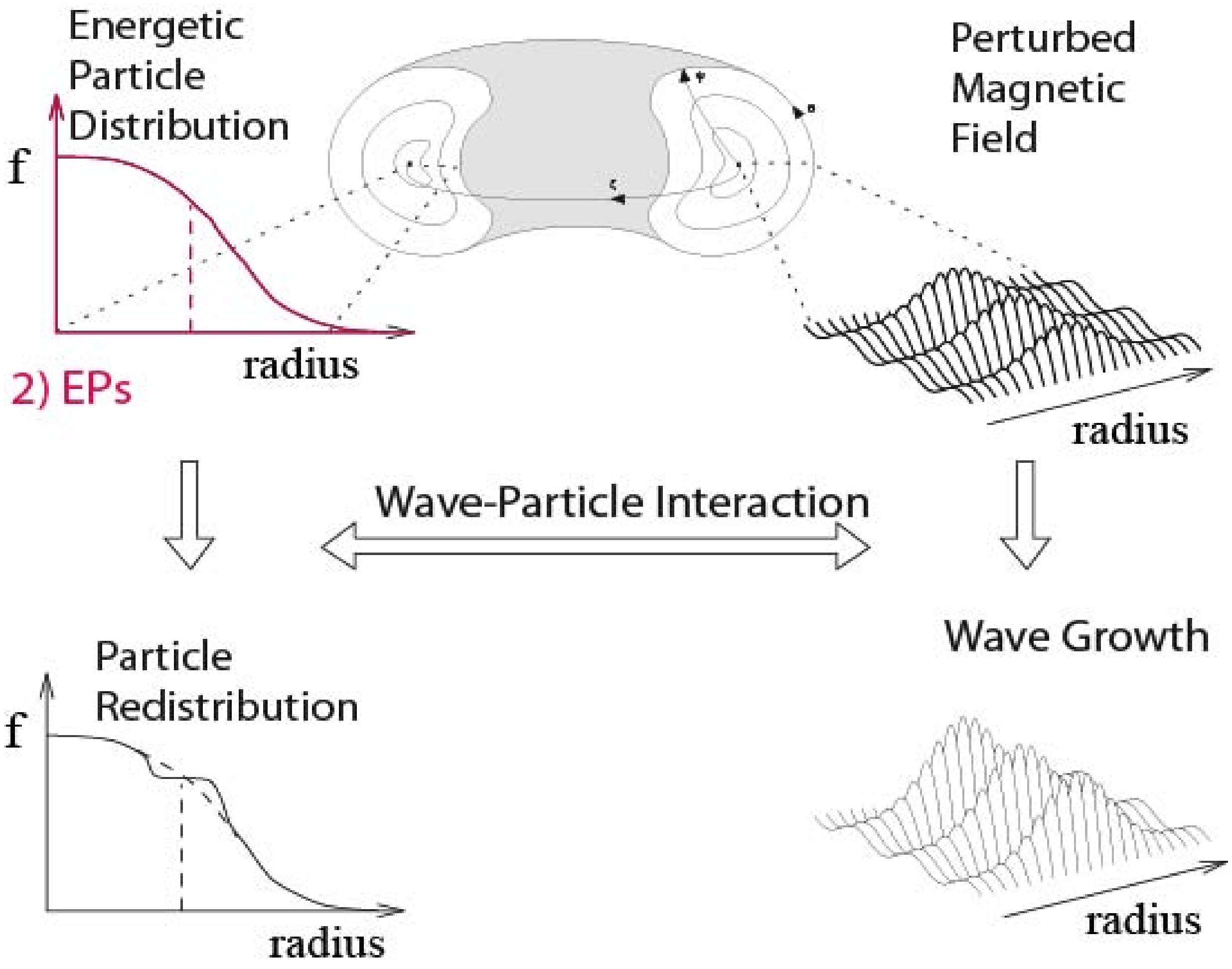
TAE (coupling)

generated by coupling of m = 5,6 (n = 2)

Soft X-ray Tomogr.

Weller, Phys. Plasmas 8 (2001); PRL 72 (1994)

Part 2: Energetic Particles


Energetic Particle Distribution

2) EPs

Perturbed Magnetic Field

Wave-Particle Interaction

Particle Redistribution

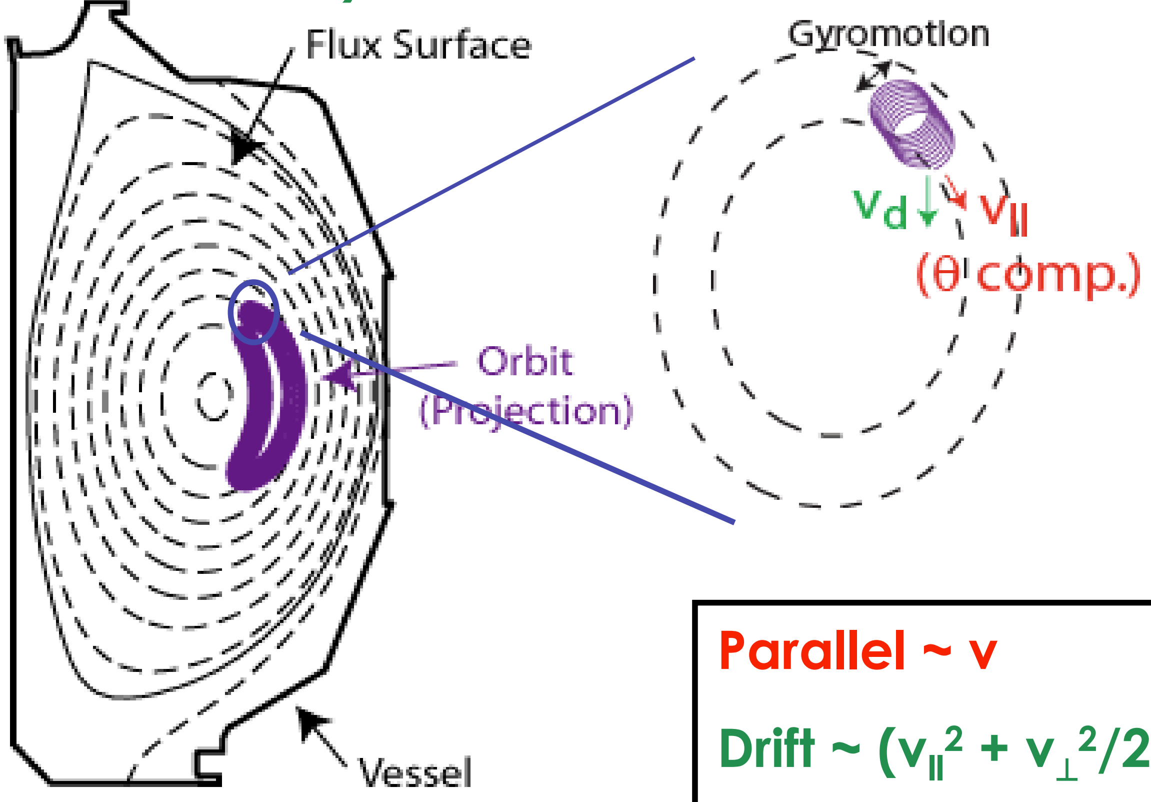
Wave Growth

1. Alfvén Gap Modes
2. **Energetic Particles (EP)**
3. **Energetic Particle Modes (EPM)**
4. **Nonlinear Dynamics**
5. **Prospects for Control**

Pinches, Ph.D. Thesis

Fast-ion orbits have large excursions from magnetic field lines

Elevation (80 keV D^+ ion in DIII-D)



- Perp. velocity → gyromotion
- Parallel velocity → follows flux surface
- Curvature & Grad B drifts → excursion from flux surface

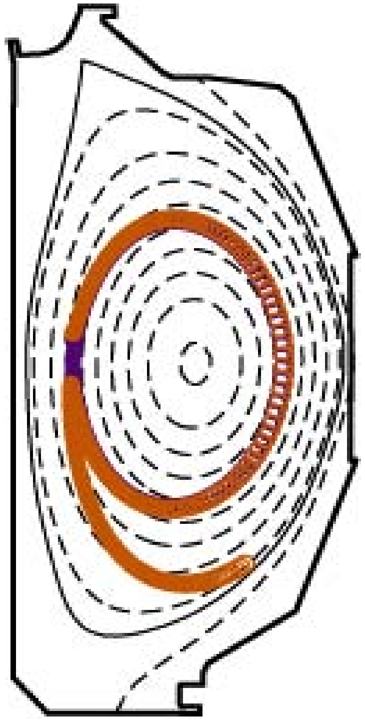
Parallel ~ v

Drift ~ $(v_{\|}^2 + v_{\perp}^2/2)$

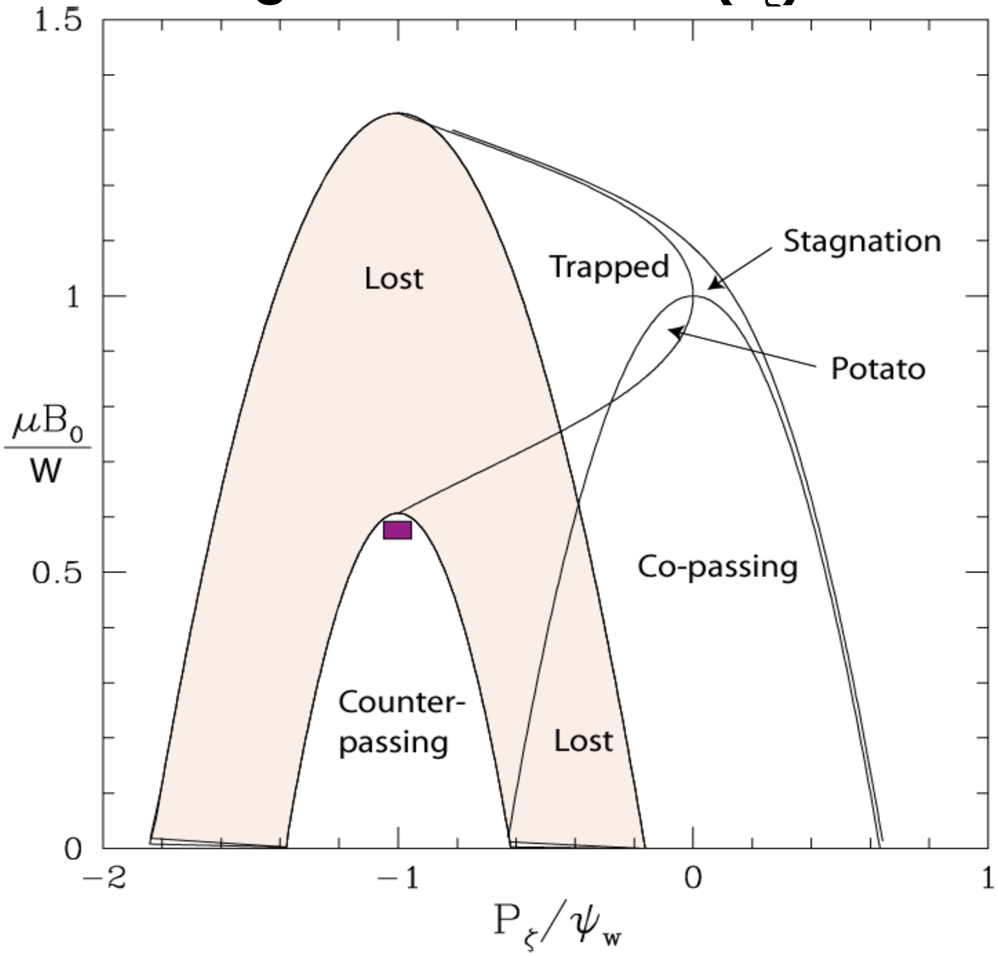
→ **Large excursions for large velocities**

Complex EP orbits are most simply described using constants of motion

Projection of 80 keV D^+ orbits in the DIII-D tokamak



Constants of motion on orbital timescale: energy (W), magnetic moment (μ), toroidal angular momentum (P_ξ)

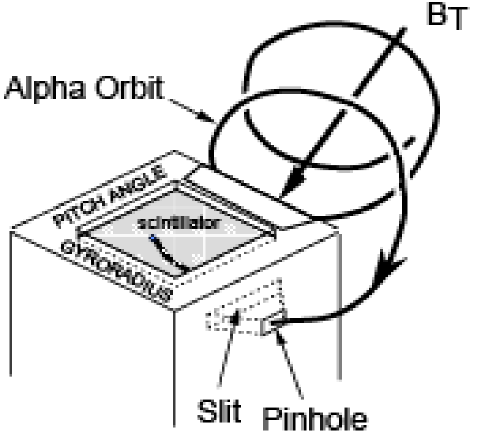


Distribution function: $f(W,\mu,P_\xi)$

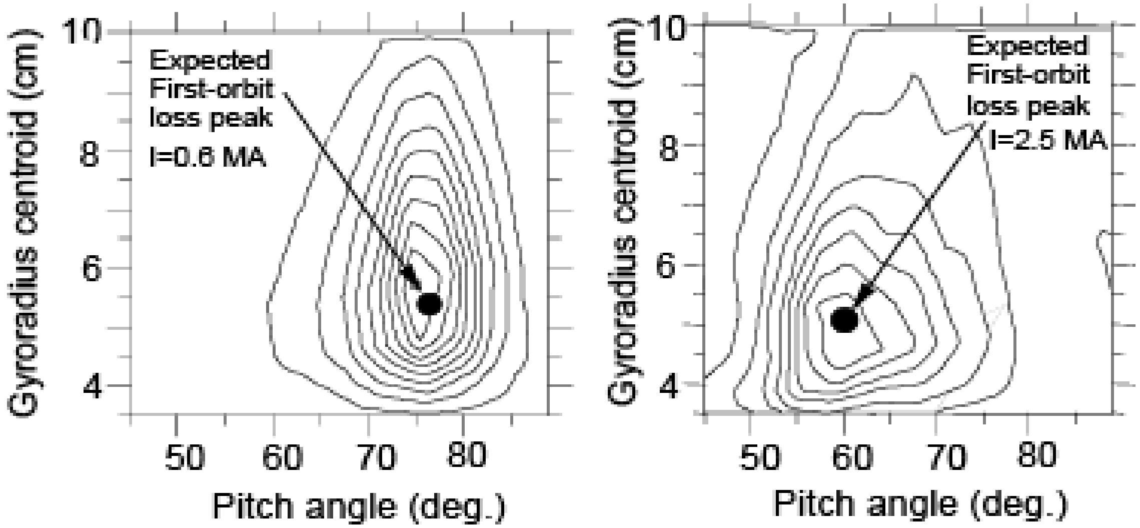
Roscoe White, Theory of toroidally confined plasmas

Orbit topology is well understood

Edge loss detector on the TFTR tokamak

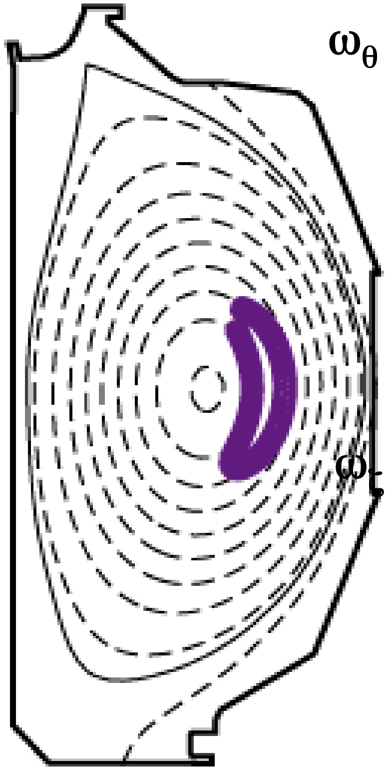


Prompt losses of D-T alpha particles to a scintillator at the bottom of TFTR



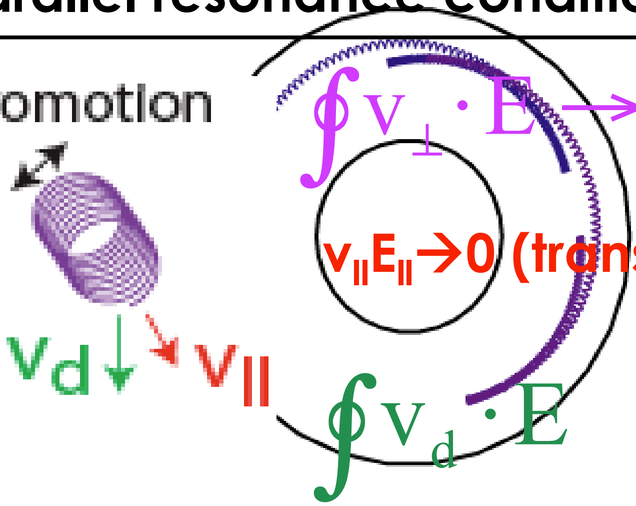
Zweben, Nucl. Fusion 40 (2000) 91

The drift motion must resonate with a wave harmonic to exchange net energy



Parallel resonance condition: $\omega = n\omega_\zeta + p\omega_\theta$

Time to comple poloida → ω_θ



$\oint v_\perp \cdot E \to 0 \quad (\Omega_c >> \omega)$

Time to complete toroidal orbit → ω_ζ

$v_\parallel E_\parallel \to 0$ (transverse polarization)

$\oint v_d \cdot E$ (main energy exchange)

Write v_d as a Fourier expansion in terms of poloidal angle θ: $\sum_{l=\pm1,\pm2,\ldots} A_l e^{il\theta}$

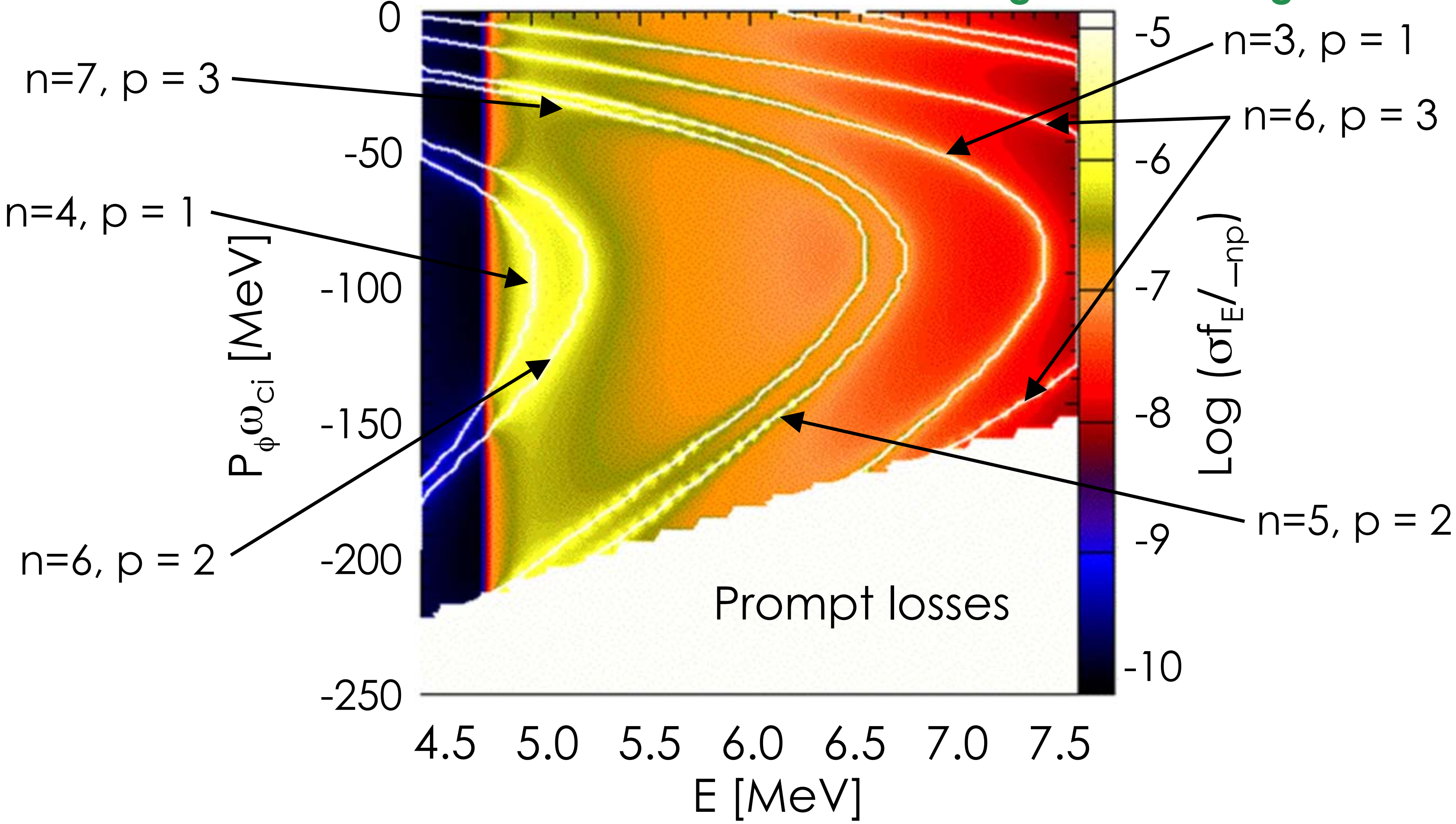
Energy exchange resonance condition: $\omega - n\omega_\zeta + (m+l)\omega_\theta = 0$

Wave mode #s

Drift harmonic

A typical distribution function has many resonances

Calculated resonances with observed TAEs during RF ion heating in JET

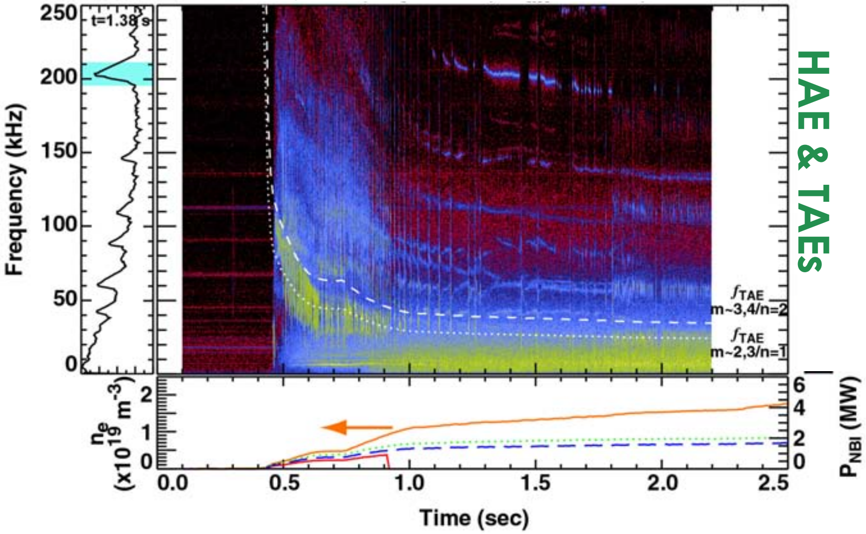


- Resonance condition, $\_{np} = n\,\_{\zeta} - p\,\_{\_} - \_ = 0$

Pinches, Nucl. Fusion 46 (2006) S904

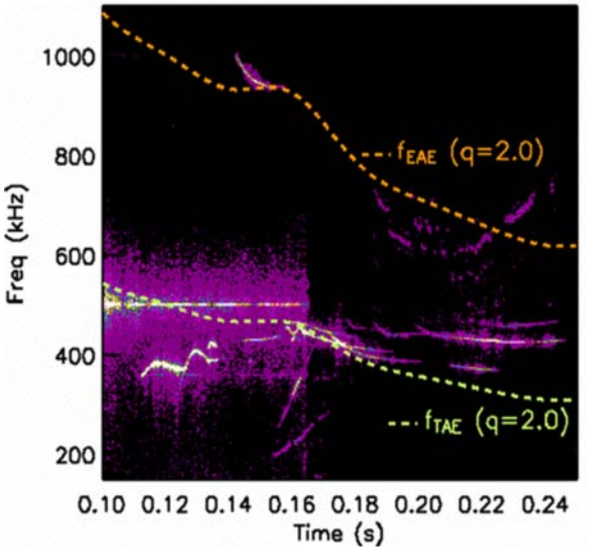
Tremendous variety of resonances are observed

Beam injection, CHS stellarator[1]



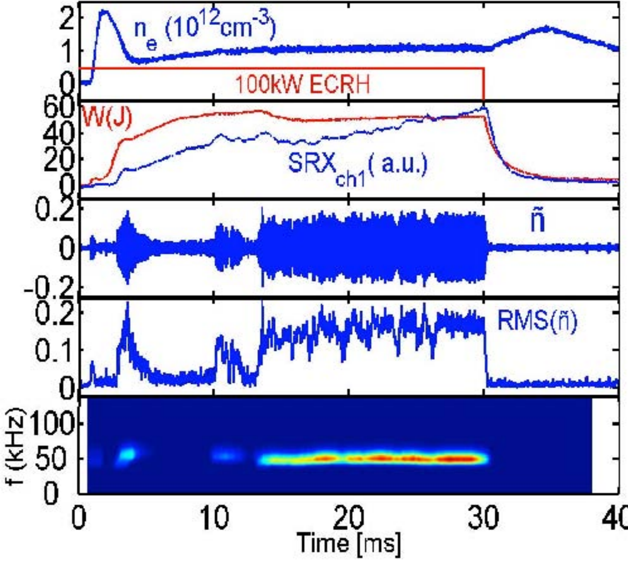
HAE & TAEs

RF tail ions, C-Mod tokamak[2]



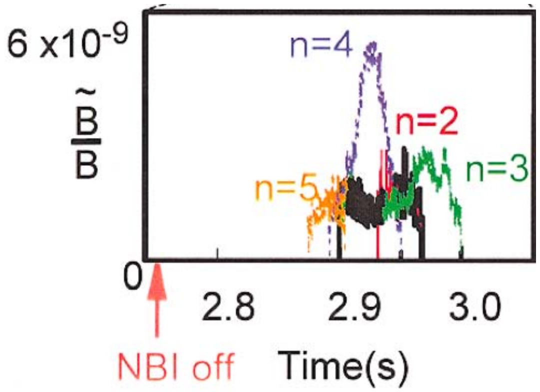
EAEs, TAEs, & RSAEs

Electron tail, HSX stellarator[3]



GAE

Alphas, TFTR tokamak[4]



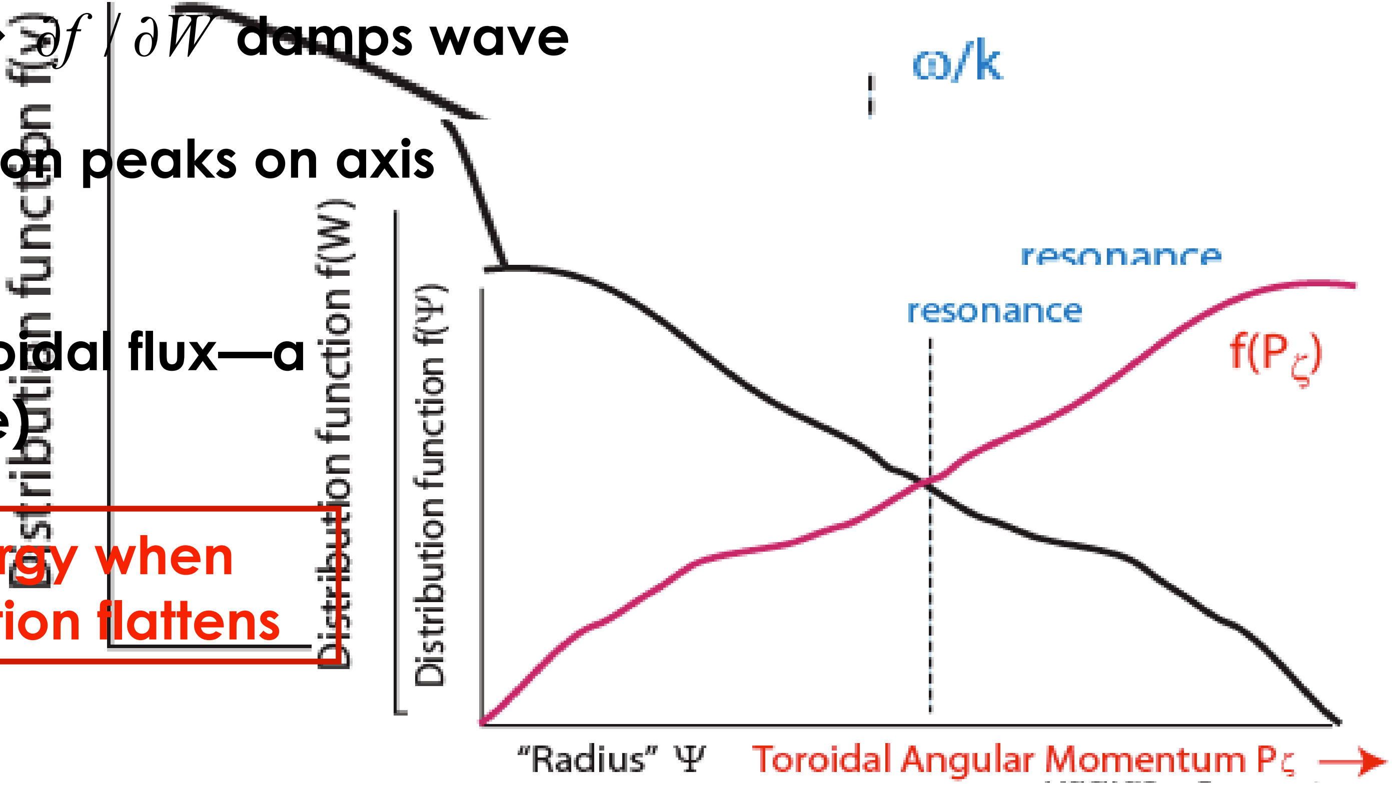
TAEs & RSAEs

[1]Yamamoto, PRL 91 (2003) 245001; [2]Snipes PoP 12 (2005) 056102; [3]Brower; [4]Nazikian, PRL 78 (1997) 2976

The spatial gradient of the distribution usually drives instability

- Slope of distribution function at resonances determines whether particles damp or drive instability
- If $\gamma \; \partial f / \partial v < 0$ $\omega \partial f / \partial W + n \partial f / \partial P_\zeta$ the wave damps
- Energy distribution usually decreases monotonically → $\partial f / \partial W$ damps wave

Landau damping



- Spatial distribution peaks on axis
- $P_\zeta = mRv_\zeta - Ze\Psi$

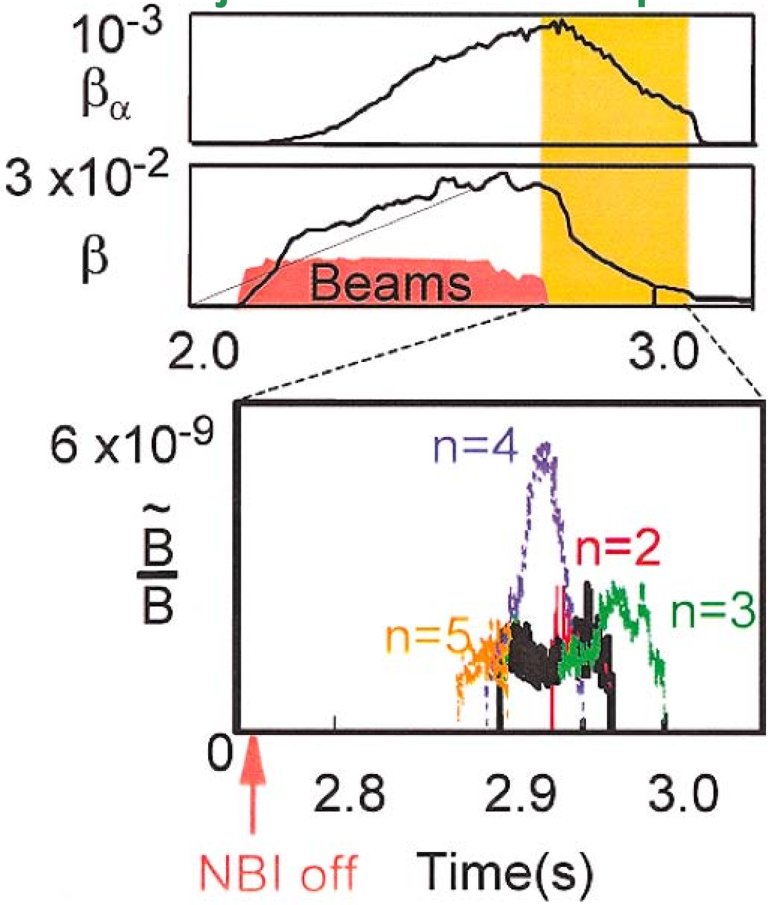
($\Psi = RA_\zeta$ is the poloidal flux—a radial coordinate)

Wave gains energy when distribution function flattens

TAEs in TFTR: avoid energy damping by beam ions, use spatial gradient drive by alphas

- Strong $\partial f / \partial W$ beam-ion damping stabilized AEs during beam pulse
- Theory[1] suggested strategy to observe alpha-driven TAEs
- Beam damping decreased faster than alpha spatial gradient drive after beam pulse
- TAEs observed[2] when theoretically predicted

AEs observed after beam injection in TFTR D-T plasmas



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[1] Fu, Phys. Plasma 3 (1996) 4036; Spong, Nucl. Fusion 35 (1995) 1687

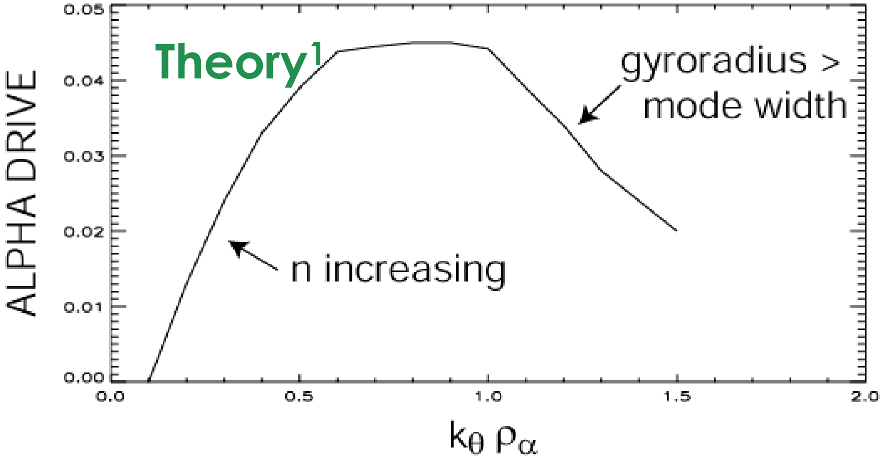
[2] Nazikian, PRL 78 (1997) 2976

EP drive is maximized for large-n modes that are spatially extended

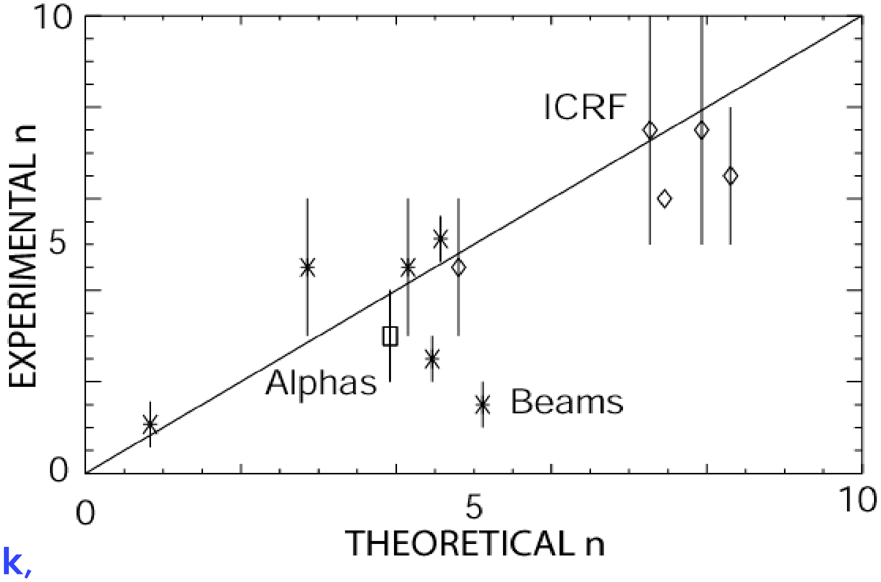
- **EP drive increases with n (stronger toroidal asymmetry)**
- **But mode size shrinks with n**
- **Weak wave-particle interaction when orbit is much larger than the mode**

→ Drive maximized when orbit width ~ mode size ($k_\theta \rho_{EP}$~ 1)

→ Large n anticipated in reactors

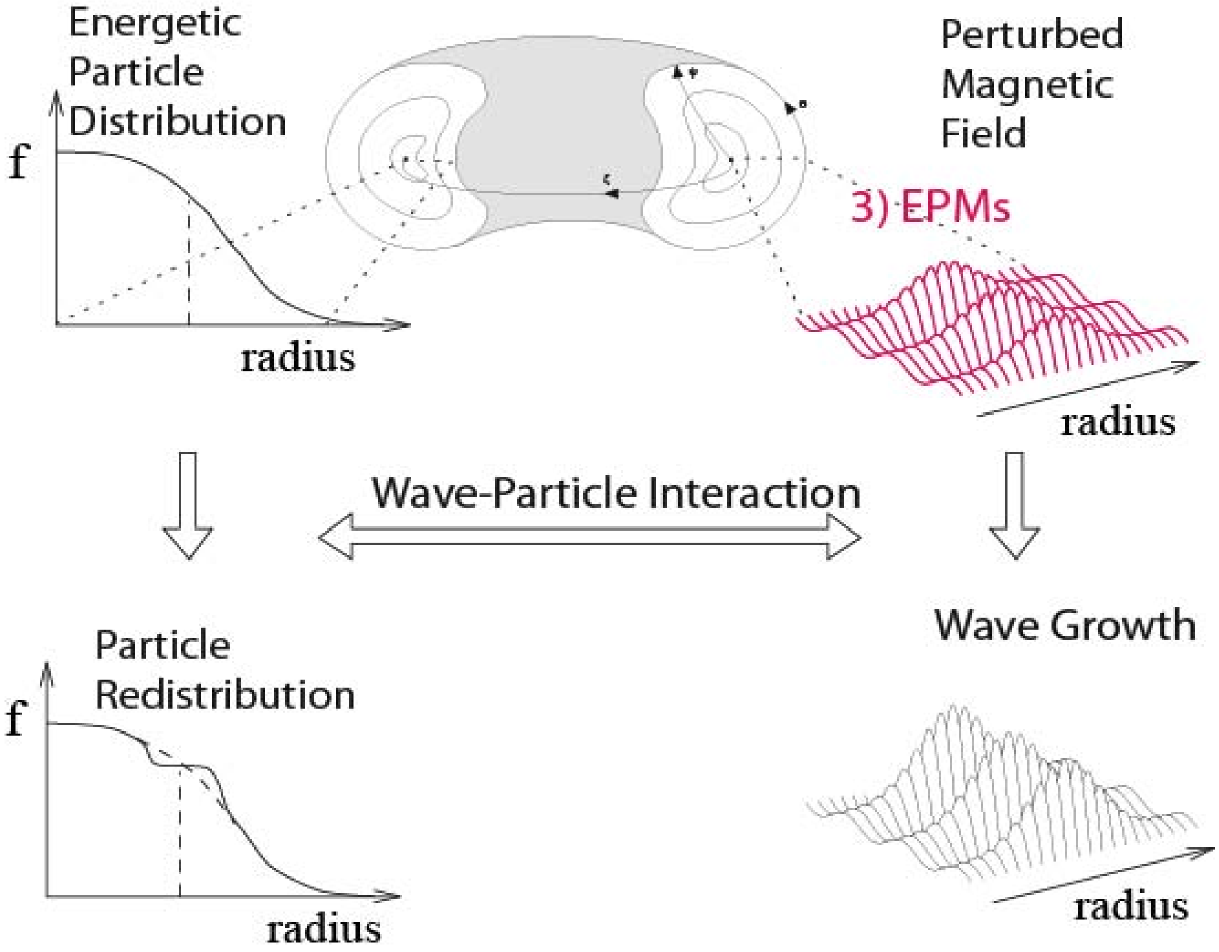


Most unstable mode number vs. theory (many tokamaks)[2]



[1] Fu, Phys. Fluids B 4 (1992) 3722; [2] Heidbrink, Pl. Phys. Cont. Fusion 45 (2003) 983

Part 3: Energetic Particle Modes (EPM)



1. Alfvén Gap Modes
2. Energetic Particles (EP)
3. **Energetic Particle Modes (EPM)**
4. **Nonlinear Dynamics**
5. **Prospects for Control**

Pinches, Ph.D. Thesis

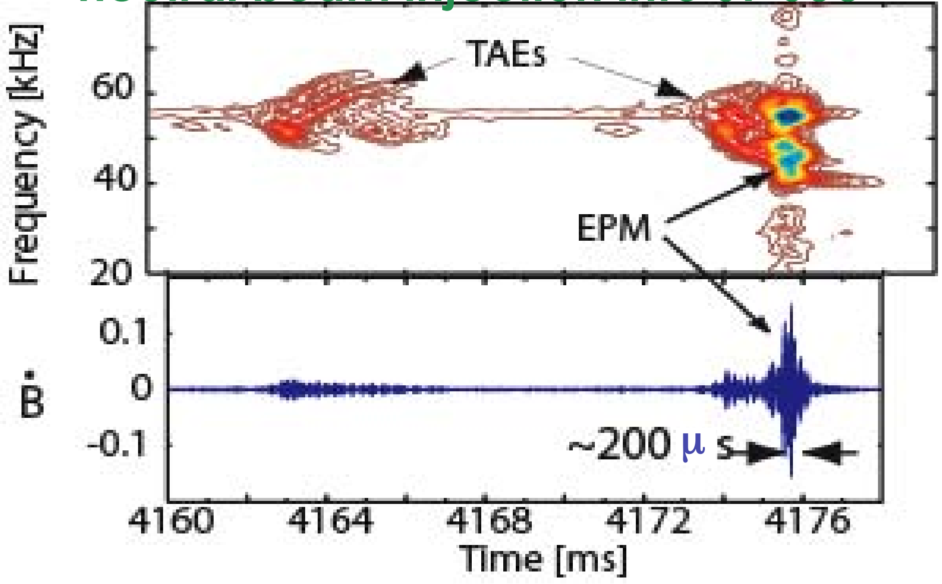
EPMs are a type of “beam mode”

Normal Mode (gap mode)	Energetic Particle Mode[1]
$n_{EP} \ll n_e$	$\beta_{EP} \sim \beta$
Wave exists w/o EPs.	EPs create a new wave branch
Re(ω) unaffected by EPs.	Re(ω) depends on EP distrib. function
EPs resonate with mode, altering Im(ω)	EPs resonate with mode, altering Im(ω)
Gap mode avoids continuum damping	Intense drive overcomes continuum damping

Chen, Phys. Plasma 1 (1994) 1519

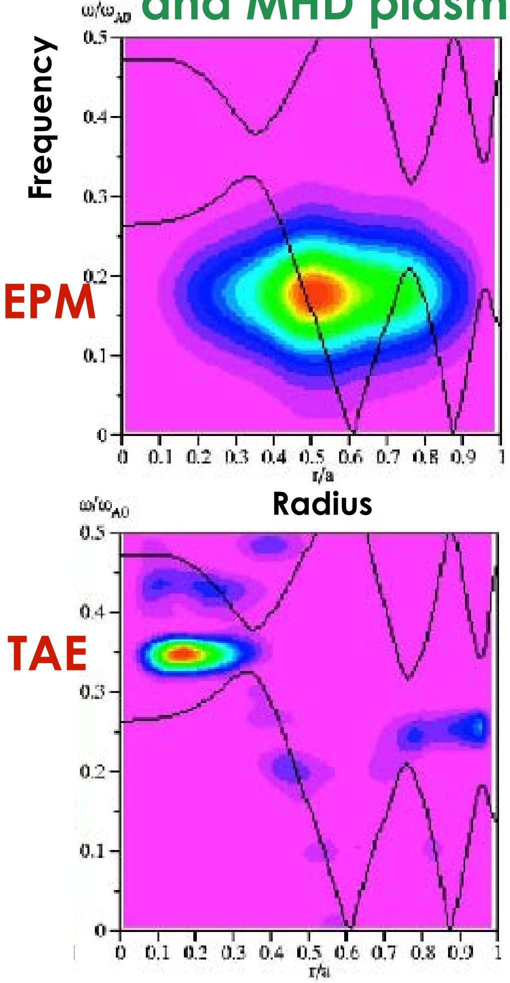
EPMs often sweep rapidly in frequency as distribution function changes

Modes observed during intense negative neutral beam injection into JT-60U



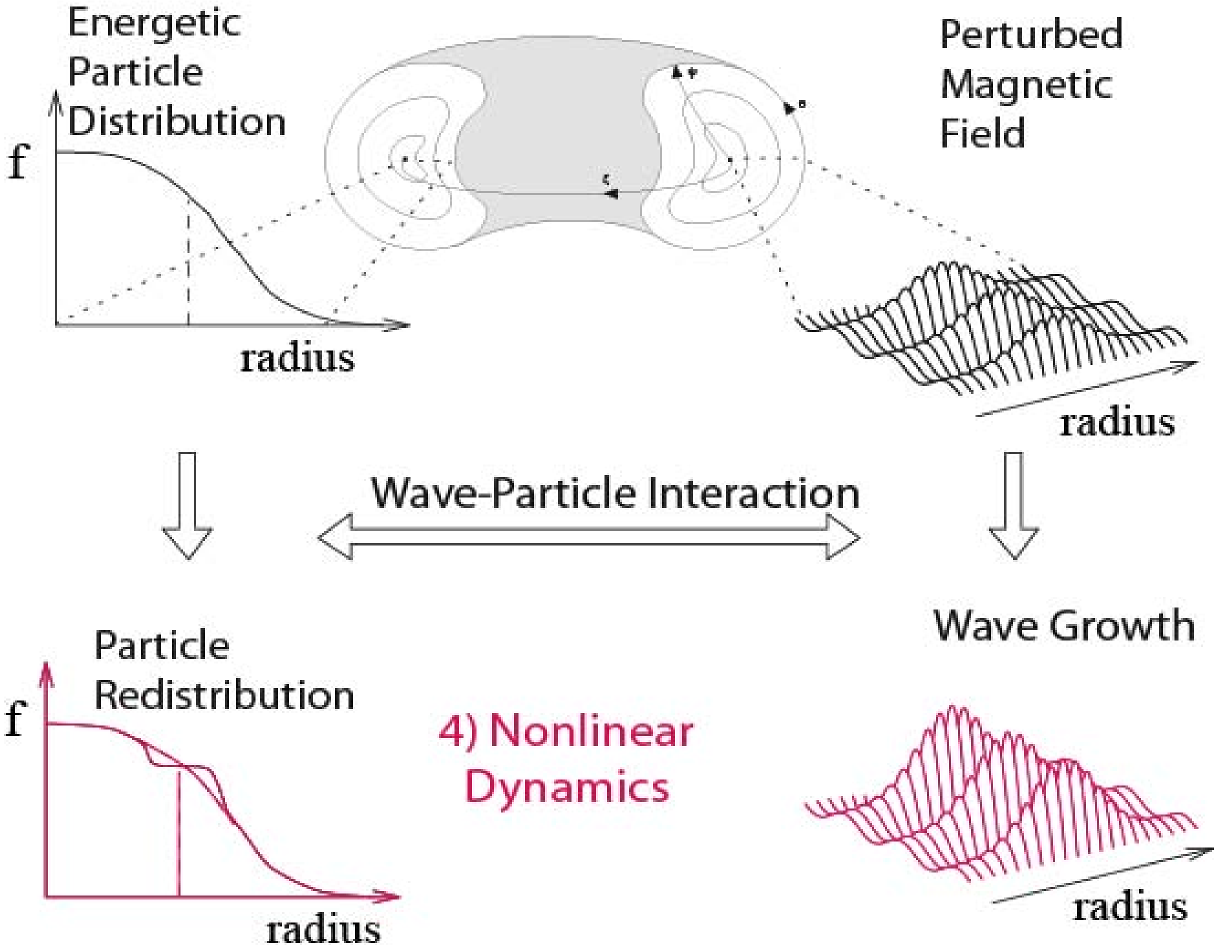
Shinohara, Nucl. Fusion 41 (2001) 603

Simulation with kinetic fast ions and MHD plasma



Briguglio, Phys. Pl. 14 (2007) 055904

Part 4: Nonlinear Dynamics



Energetic Particle Distribution

Perturbed Magnetic Field

Wave-Particle Interaction

Wave Growth

Particle Redistribution

4) Nonlinear Dynamics

Pinches, Ph.D. Thesis

1. Alfvén Gap Modes
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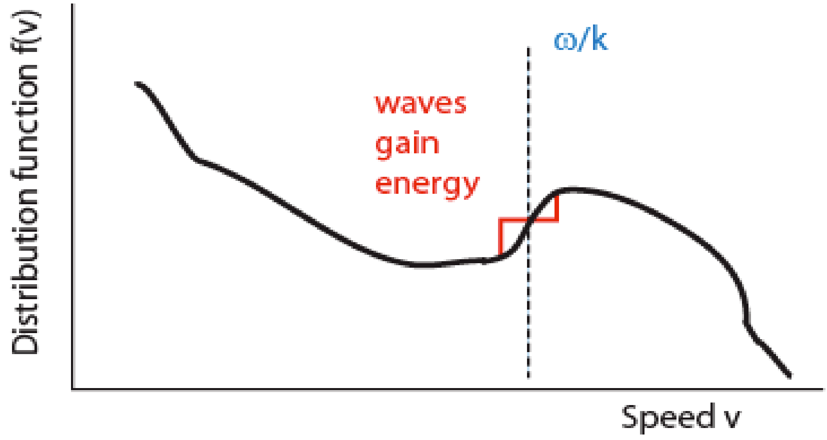
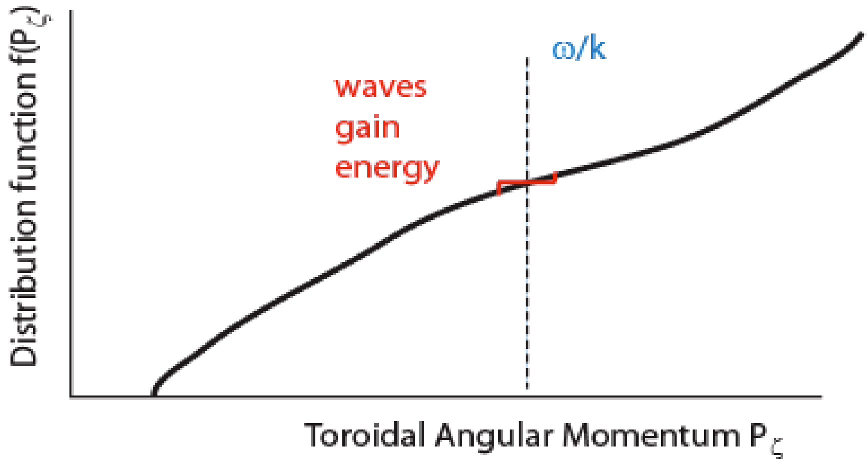
5. **Prospects for Control**

1D analogy to electrostatic wave-particle trapping describes many phenomena





- **Analogy between “bump-on-tail” and fast-ion modes:**

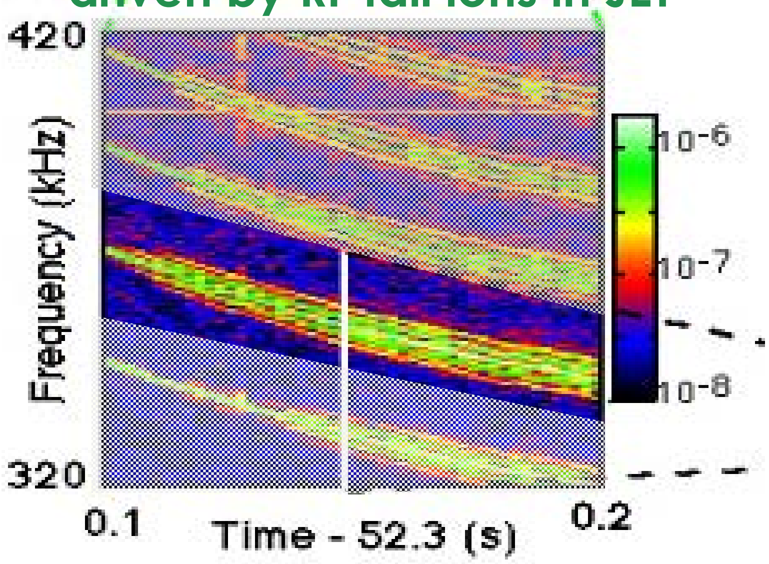
velocity-space gradient ←→

configuration-space gradient

Berk, Phys. Pl. 6 (1999) 3102

Striking Success of Berk-Breizman Model

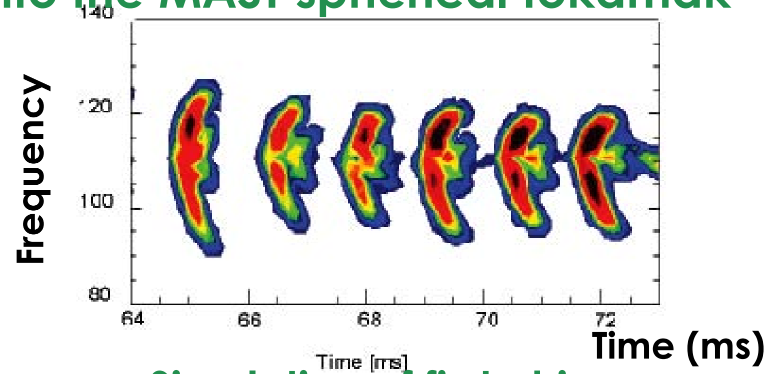
Nonlinear splitting of TAEs driven by RF tail ions in JET



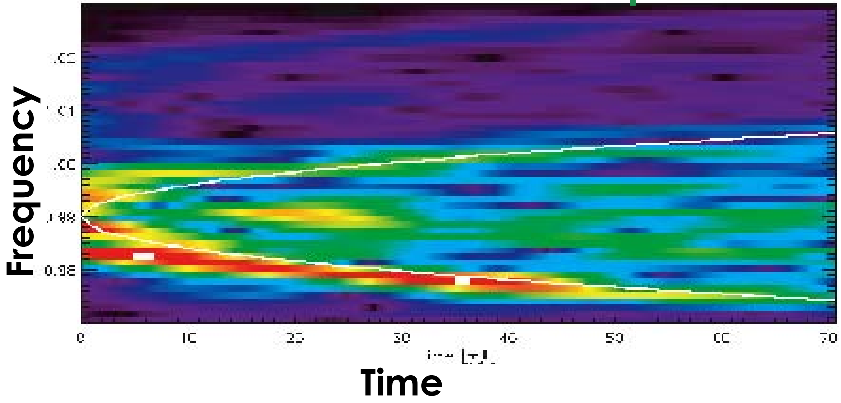
Fasoli, PRL 81 (1998) 5564

Appreciable ν_{eff}

Chirping TAEs during beam injection into the MAST spherical tokamak



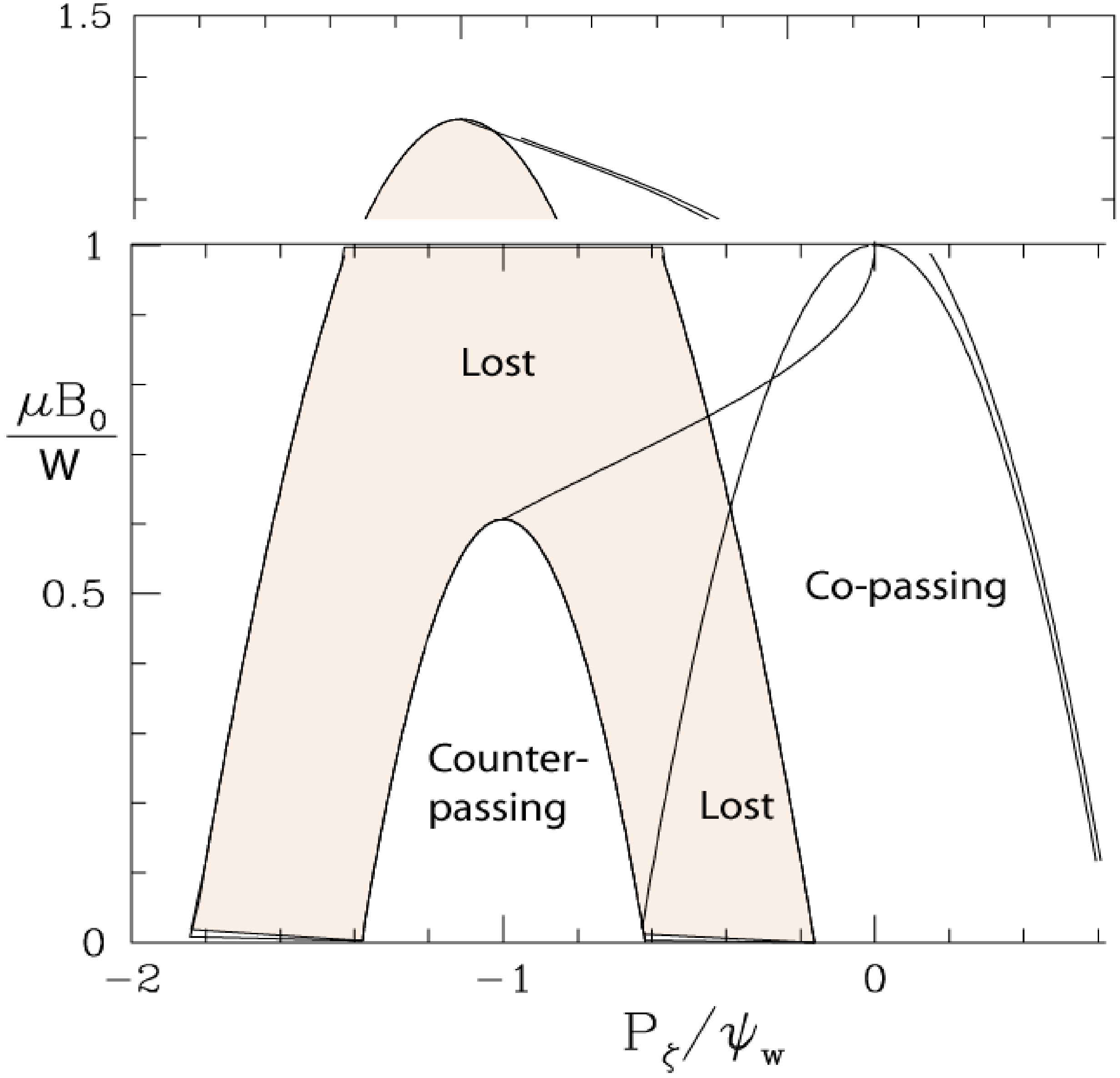
Simulation of first chirp



Pinches, Plasma Phys. Cont. Fusion 46 (2004) S47

Small ν_{eff}

Changes in canonical angular momentum cause radial transport



- Magnetic moment conserved $(\Delta\mu = 0)$
- Energy changes less than angular momentum: $\Delta W/W \sim 0.1(\Delta P_\zeta/P_\zeta)$
- $\Delta P_\zeta \rightarrow \Delta\Psi$ (radial transport)

- Leftward motion on graph implies outward radial motion

Four mechanisms of EP transport are distinguished

1) Convective loss boundary ($\sim B_r$, small %)

Fluctuations in equilibrium push EPs across loss boundary

2) Convective phase locked ($\sim B_r$, large %)

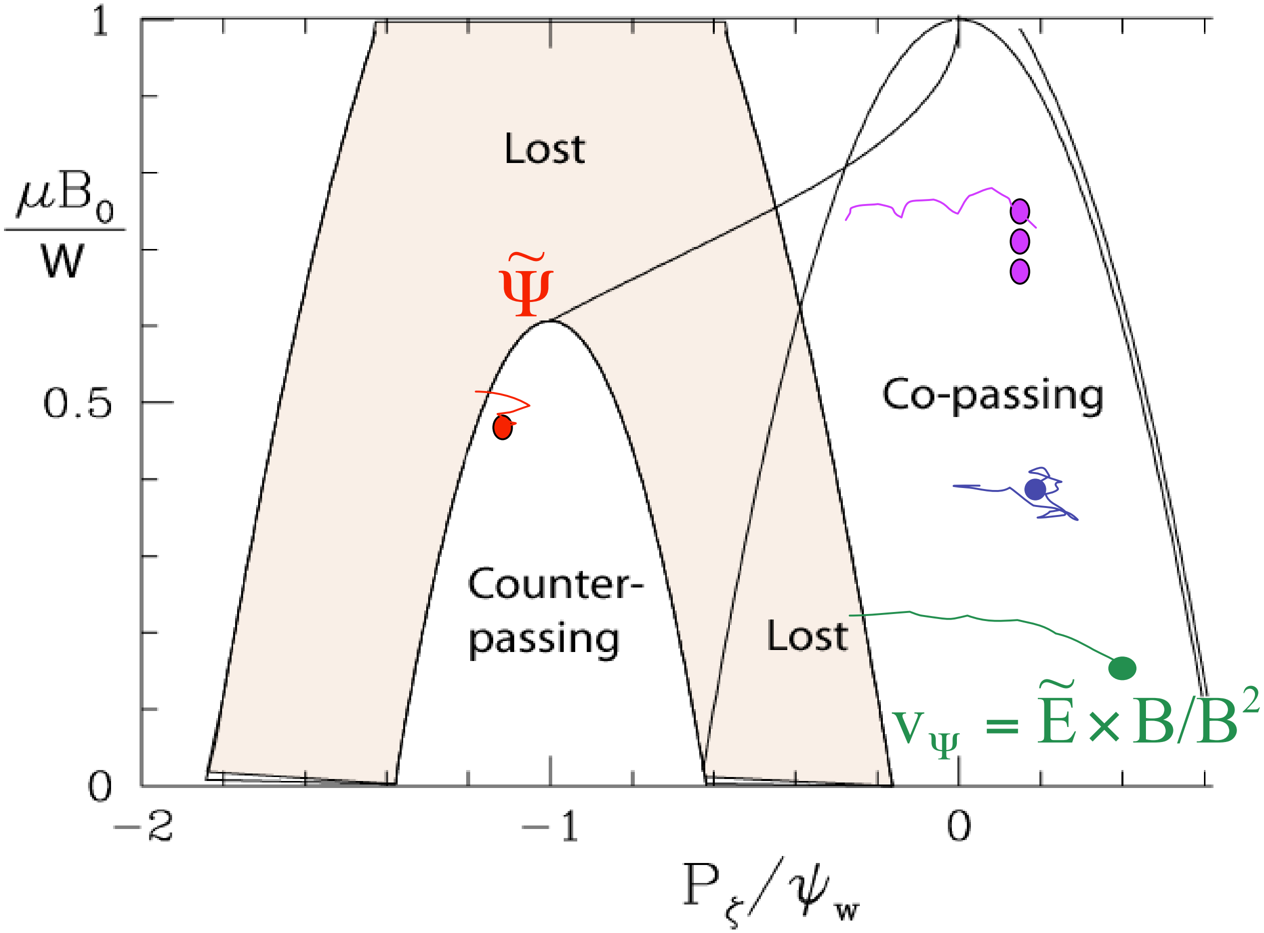
EPs stay in phase with wave as they "walk" out of plasma

3) Diffusive transport ($\sim B_r^2$)

Random walk due to multiple resonances

4) Avalanche (B_r threshold)

EP transport locally increases gradient, destabilizing new modes



1) Convective loss boundary ($\sim B_r$)

2) Convective phase locked ($\sim B_r$)

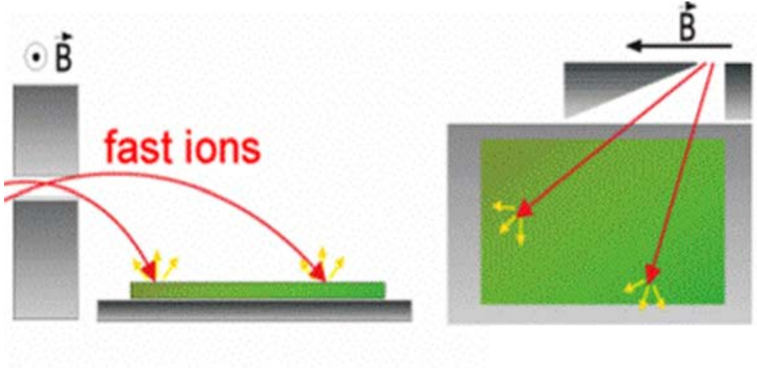
3) Diffusive transport ($\sim B_r^2$)

4) Avalanche (B_r threshold)

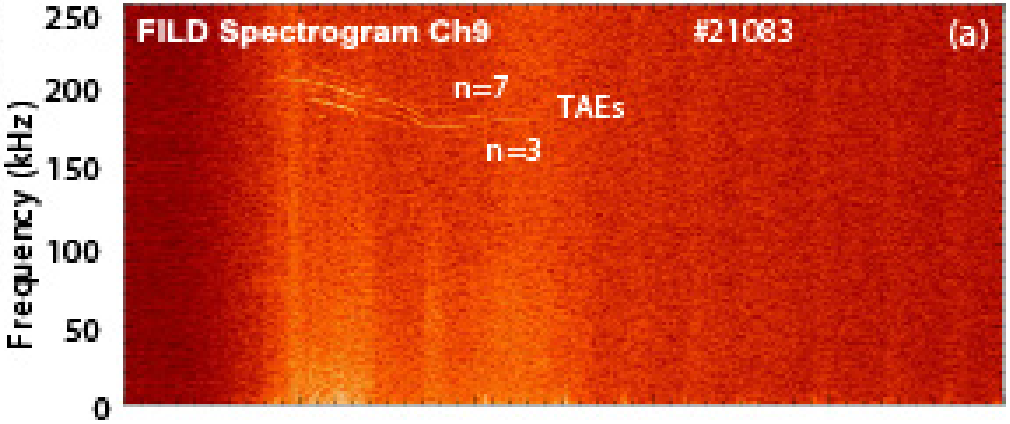
- Leftward motion on graph implies outward radial motion

Convective transport often observed

Edge scintillator on Asdex-U tokamak

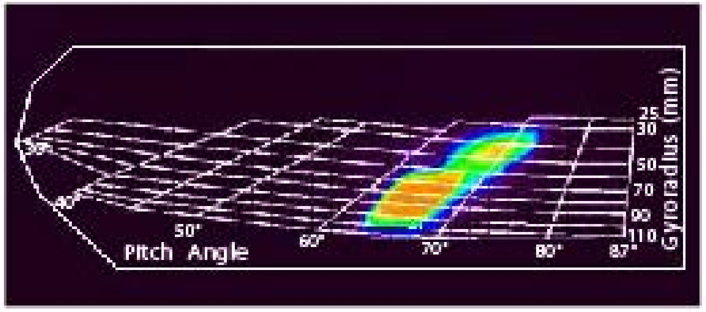


Coherent fluctuations in loss signal of RF tail ions at TAE frequencies



García-Muñoz, PRL 99 (2007) submitted

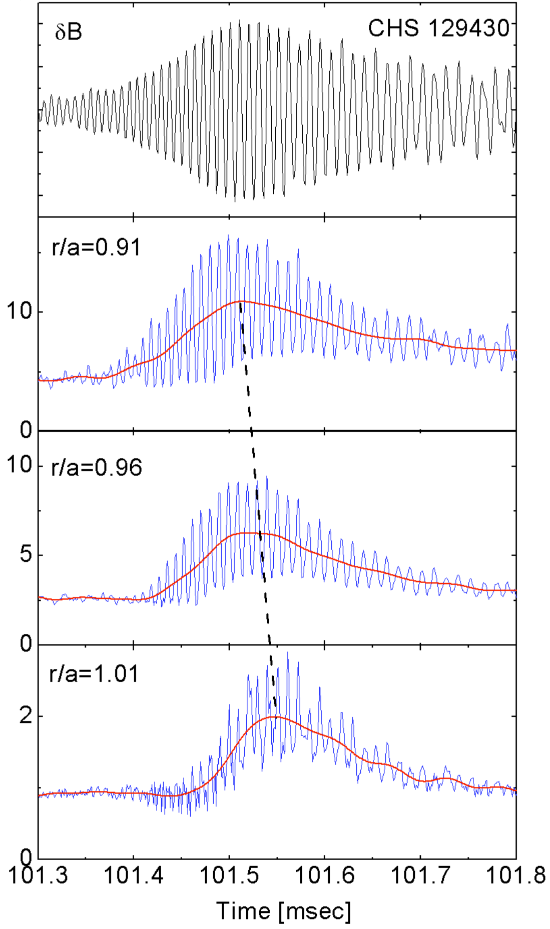
Image on scintillator screen during TAEs



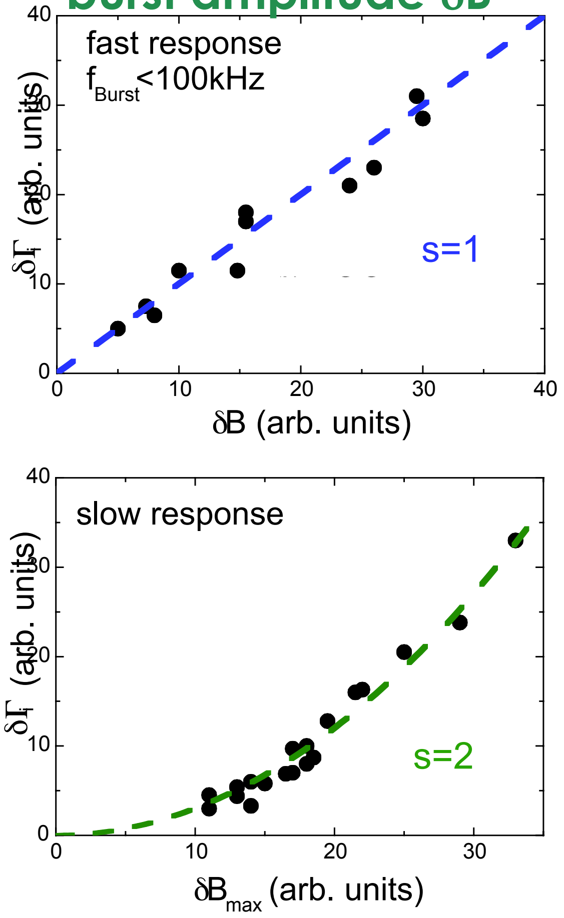
- **Fast ions cross loss boundary and hit the scintillator in phase with the waves**

Both convective and diffusive losses are observed

EPM burst & fast-ion response during beam injection into CHS stellarator



Scaling of coherent fast-ion flux and slow flux with burst amplitude δB

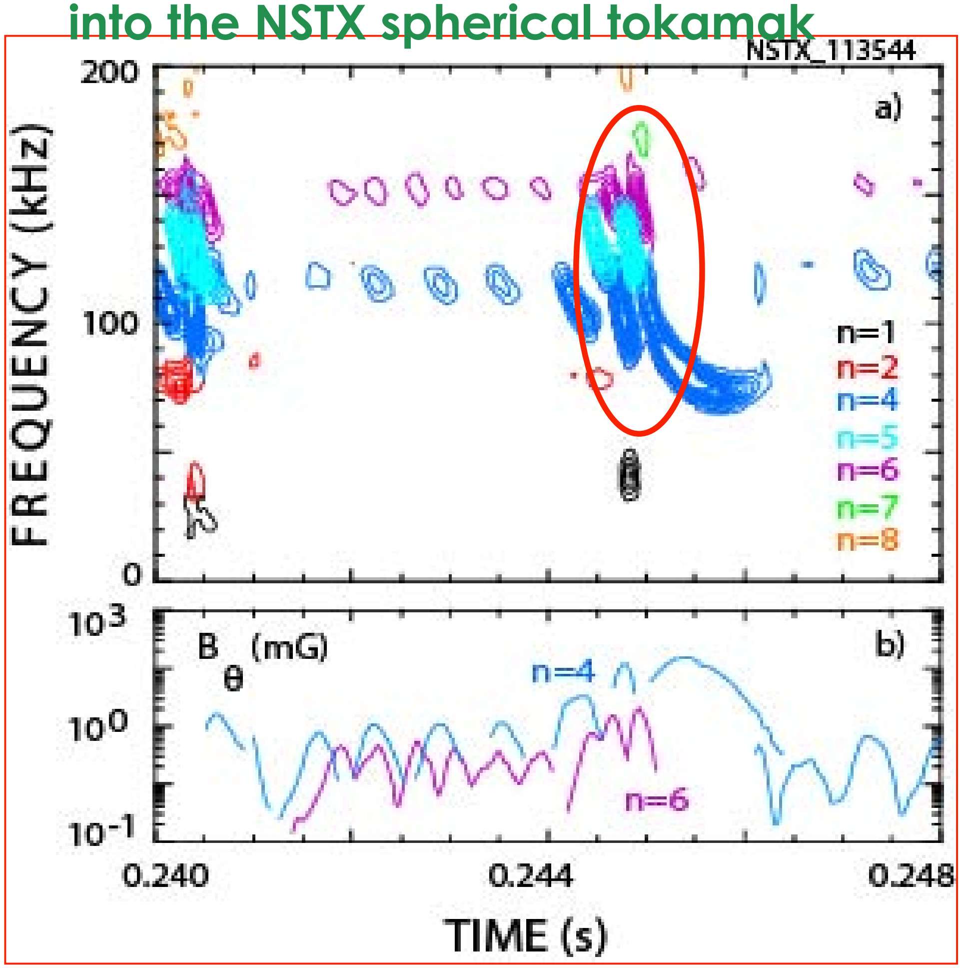


- Fast response is a resonant convective oscillation
- Slow response scales as δB^2, as expected for diffusive transport

Nagaoka (2007)

Avalanche phenomena observed

Magnetics data during beam injection into the NSTX spherical tokamak

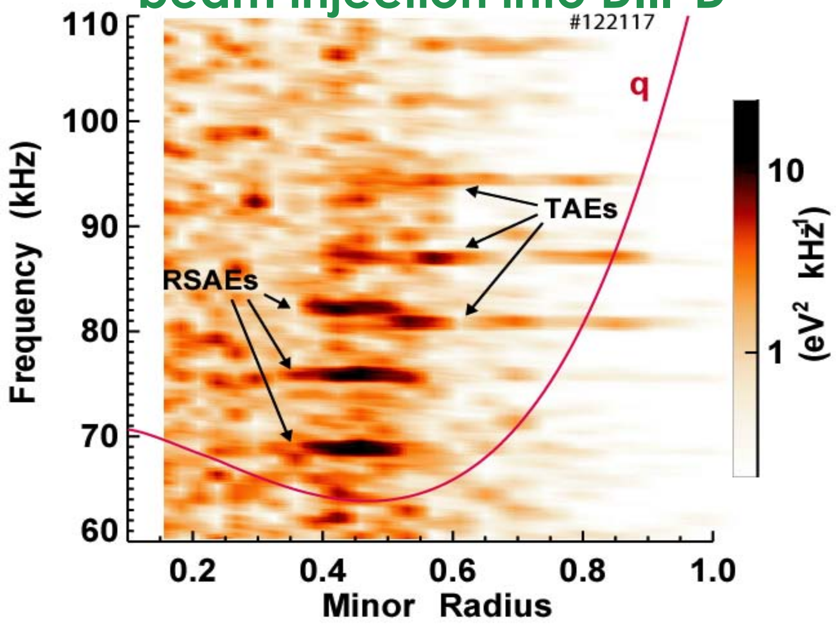


Fredrickson, Nucl. Fusion 46 (2006) S926

- When n=4 & n=6 TAE bursts exceed a certain amplitude, a large burst with many toroidal mode numbers ensues
- Fast-ion transport is much larger at avalanche events

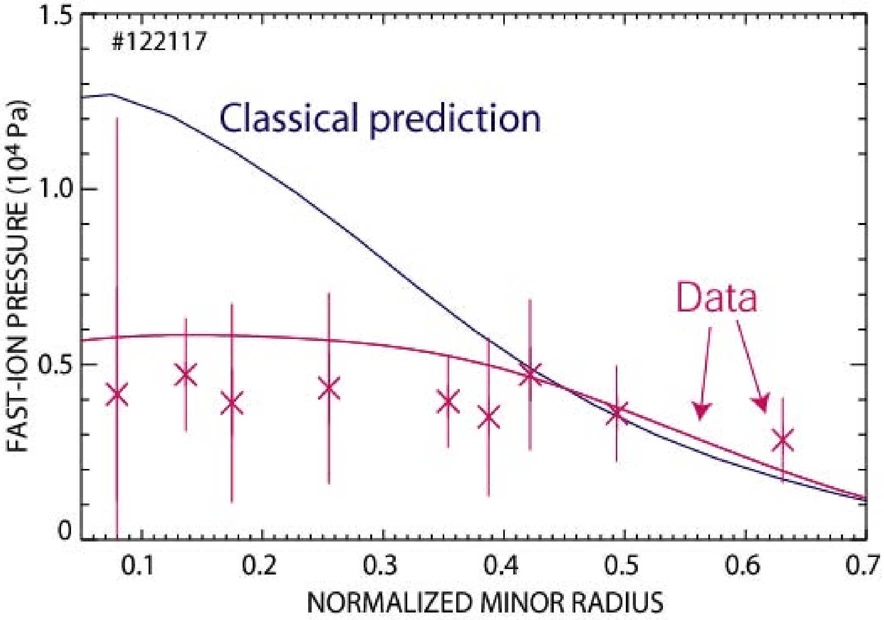
Quantitative calculations of EP transport are unsuccessful

Radial δT_e profile during beam injection into DIII-D



Van Zeeland, PRL 97 (2006) 135001

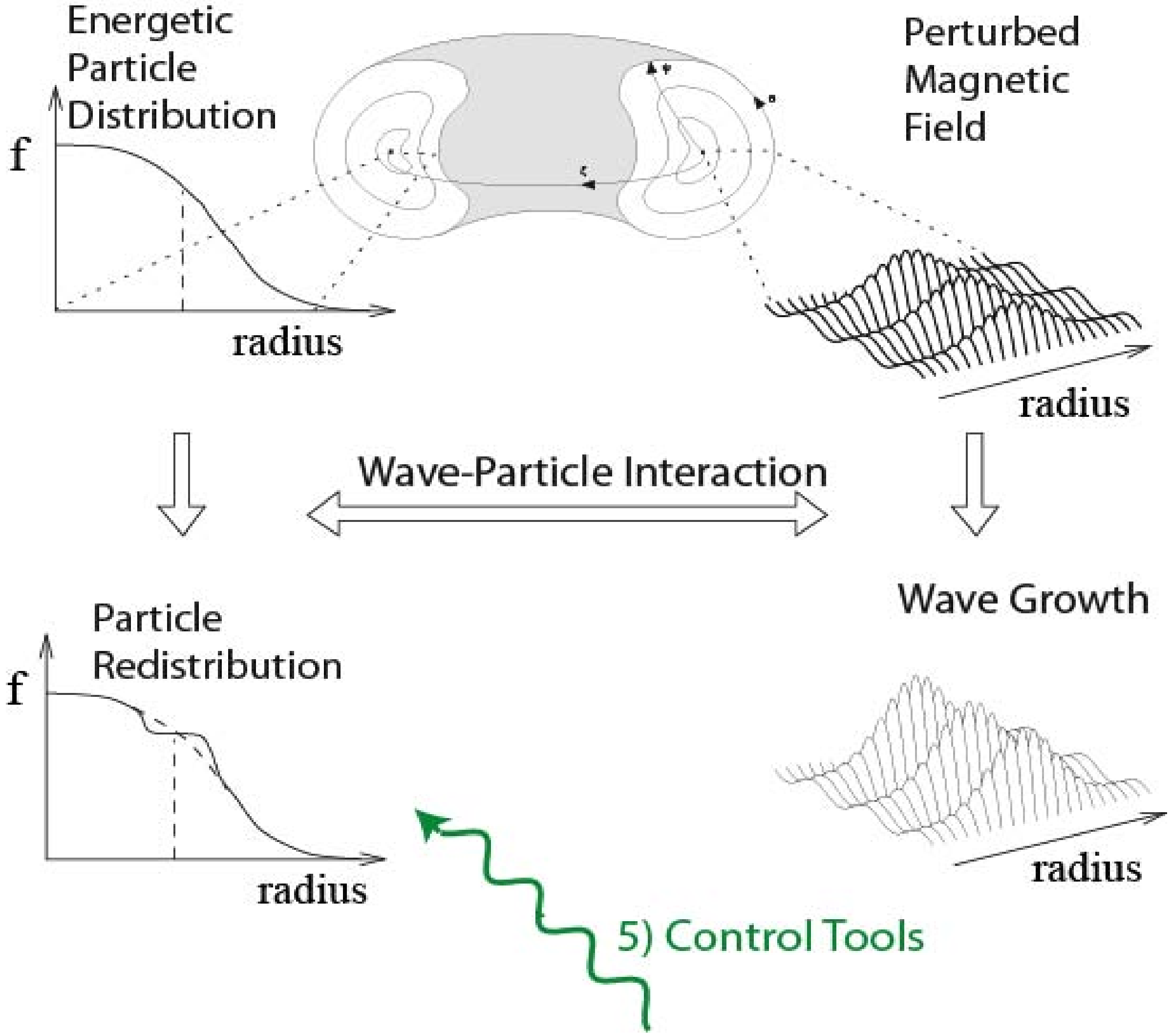
Radial fast-ion profile



Heidbrink, PRL 99 (2007) in press

- **Measured mode structure agrees well with MHD model**
- **Input these wave fields into an orbit-following code**
- **Calculate much less fast-ion transport than observed**
- **What's missing?**

Part 5: The Frontier



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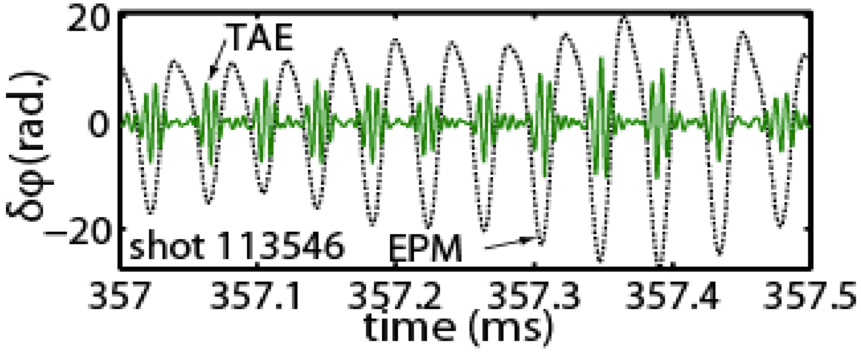
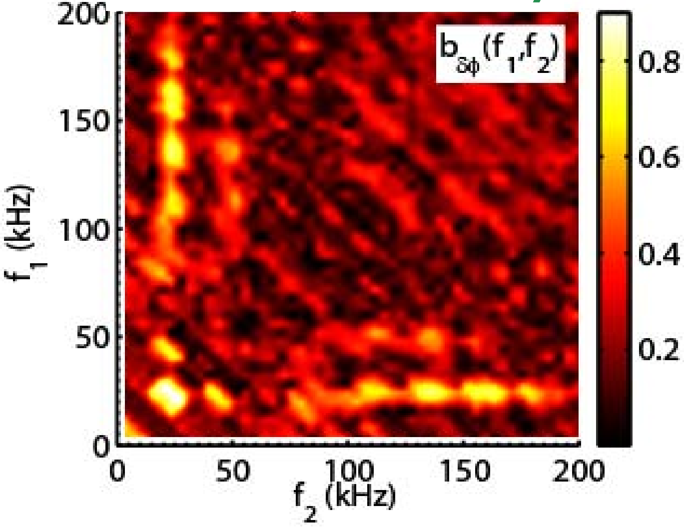
Pinches, Ph.D. Thesis

Diagnose nonlinear interactions

Filtered reflectometer δn_e signal during beam injection into NSTX



Bicoherence analysis



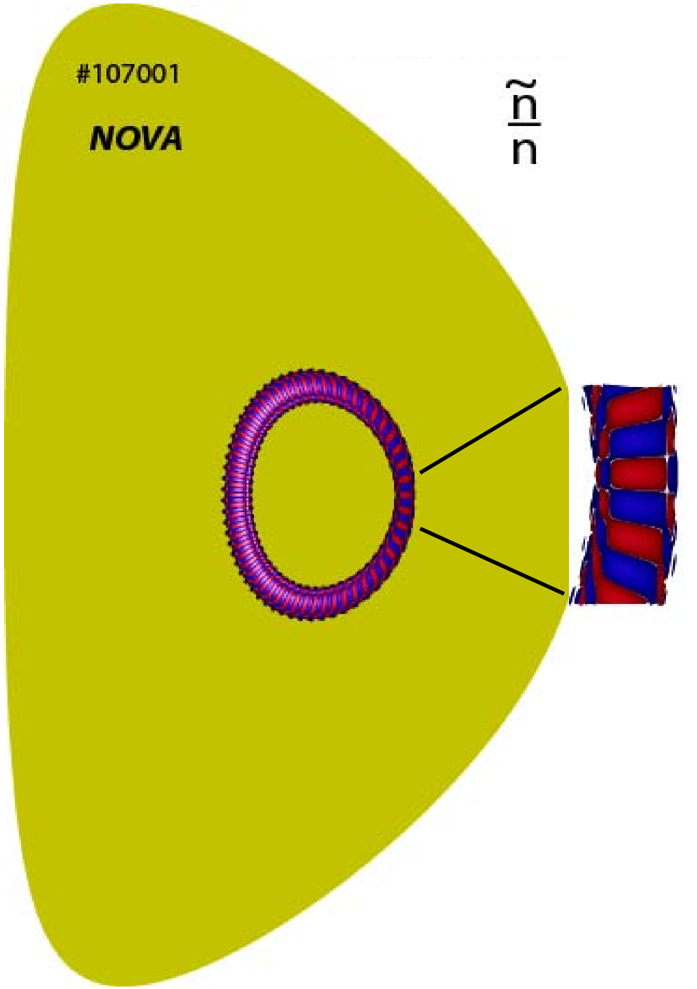
Crocker, PRL 97 (2006) 045002

APS-DPP 11/07

- This example shows that the TAEs (100-200 kHz) are nonlinearly modified by a low-frequency (~20 kHz) mode
- Similar analysis of AE wave-wave interactions and wave-particle interactions are needed

Recent observations indicate kinetic interaction with the thermal plasma

Calculated n=40 RSAE that agrees with δn_e measurements on DIII-D



- High-n modes are probably driven by thermal ions.[1]
- Alfvén modes driven by low-energy beams.[2]
- New unstable gap modes from coupling of acoustic and Alfvén waves.[3]
- Wave damping measurements that disagree with fluid plasma models.[4]
- New treatments of thermal plasma are needed

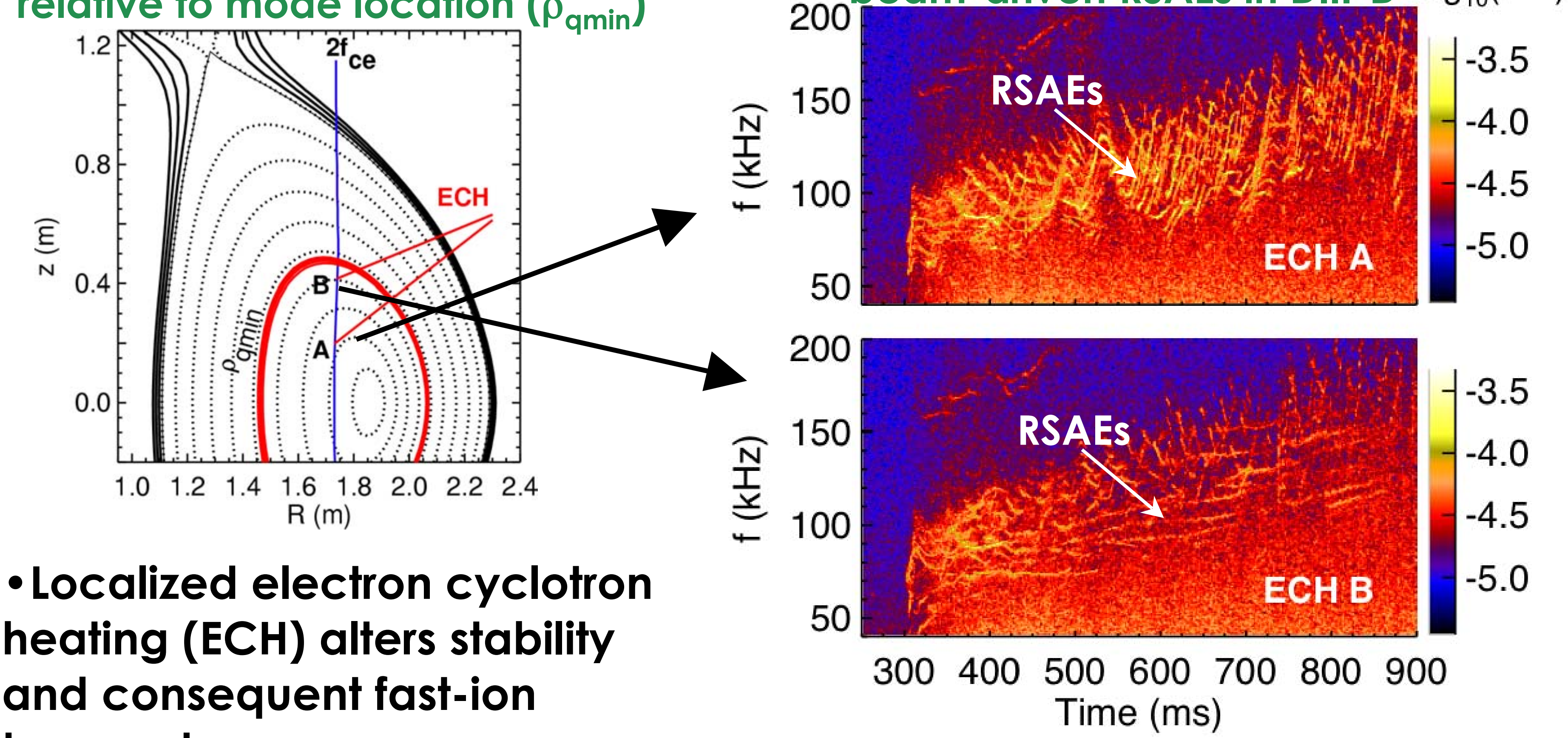
[1] *Nazikian, PRL 96 (2006) 105006; Kramer, Phys. Pl. 13 (2006) 056104*

[2] *Nazikian, JI1.01;* [3] *Gorelenkov, Phys. Lett. A 370/1 (2007) 70;* [4] *Lauber, Phys. Pl. 12 (2005) 122501*

Use control tools to alter stability

ECH deposition location is varied relative to mode location (ρ_{qmin})

Deposition near q_{min} stabilizes beam-driven RSAEs in DIII-D



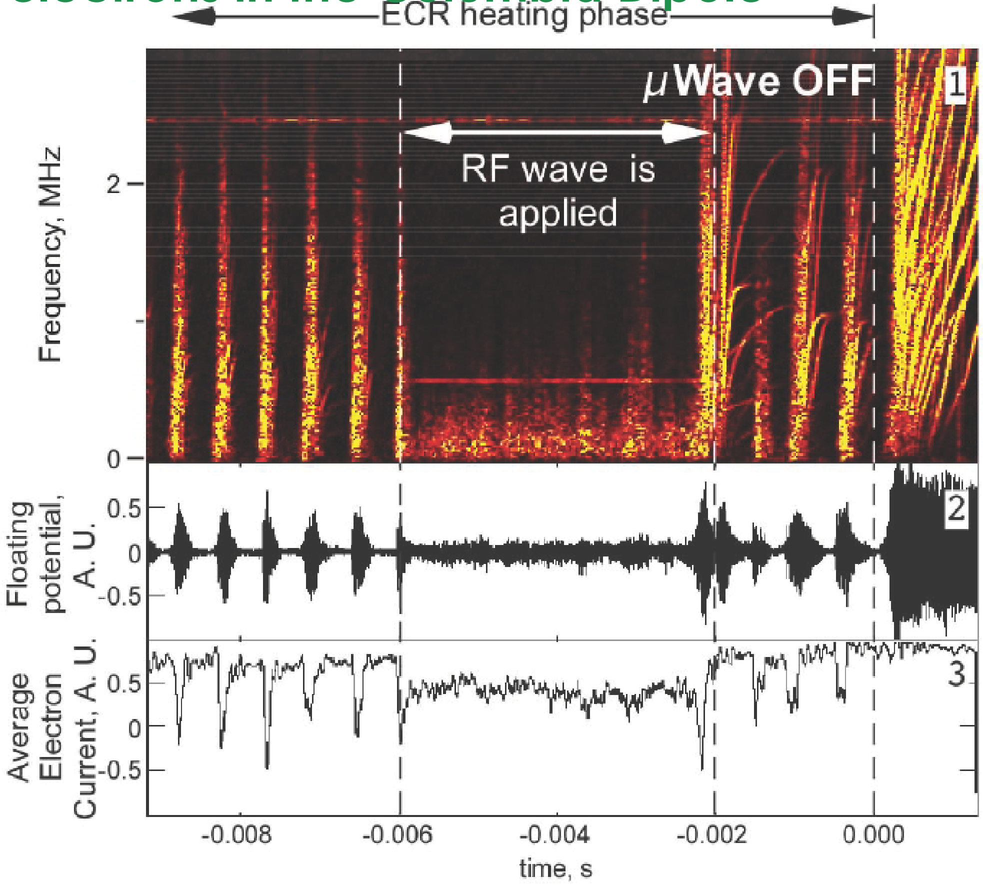
- **Localized electron cyclotron heating (ECH) alters stability and consequent fast-ion transport**

- **Can we turn off deleterious modes in a reactor?**

Van Zeeland, Plasma Phys. Cont. Fusion 49 (2007) submitted

Use control tools to alter nonlinear dynamics

Interchange instability driven by energetic electrons in the Columbia Dipole

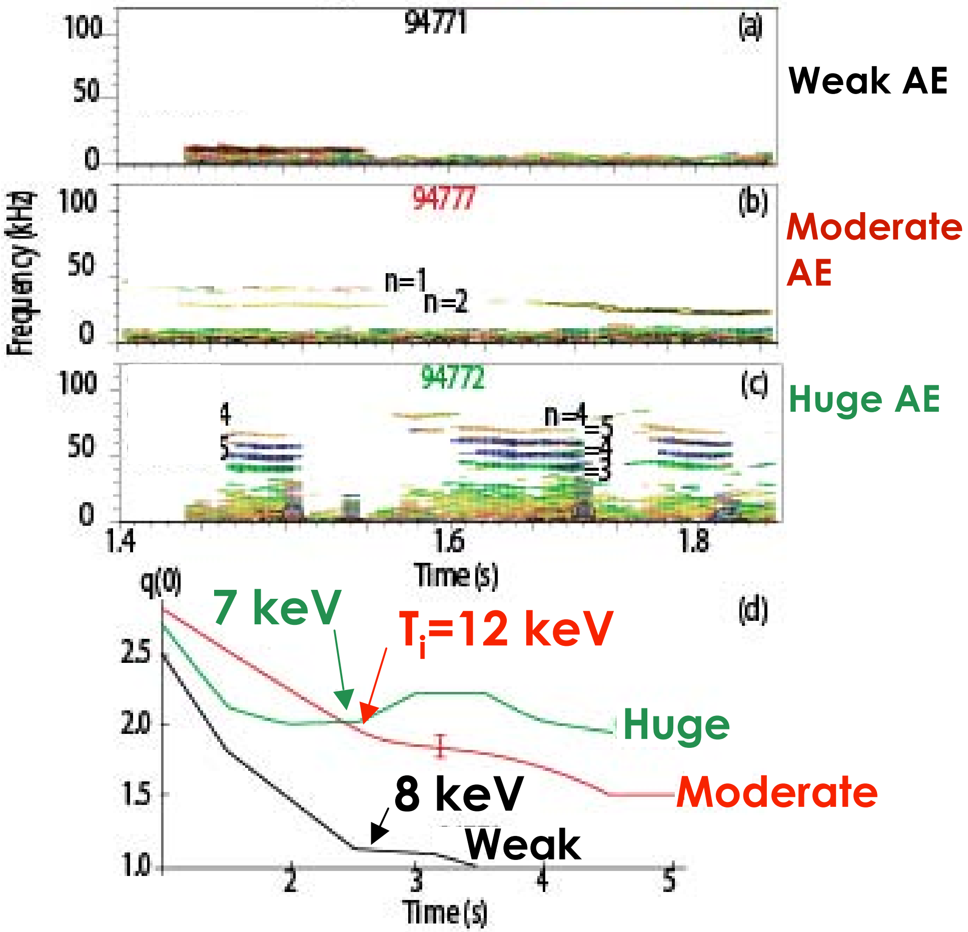


Maslovsky, Phys. Pl. 10 (2003) 1549

- In this experiment, a small amount of power (50 W) scattered EPs out of resonance, suppressing frequency chirping & eliminating large bursts
- Can we use analogous techniques to eliminate damaging bursts of lost alphas in a reactor?

Alfvén Eigenmodes can improve performance

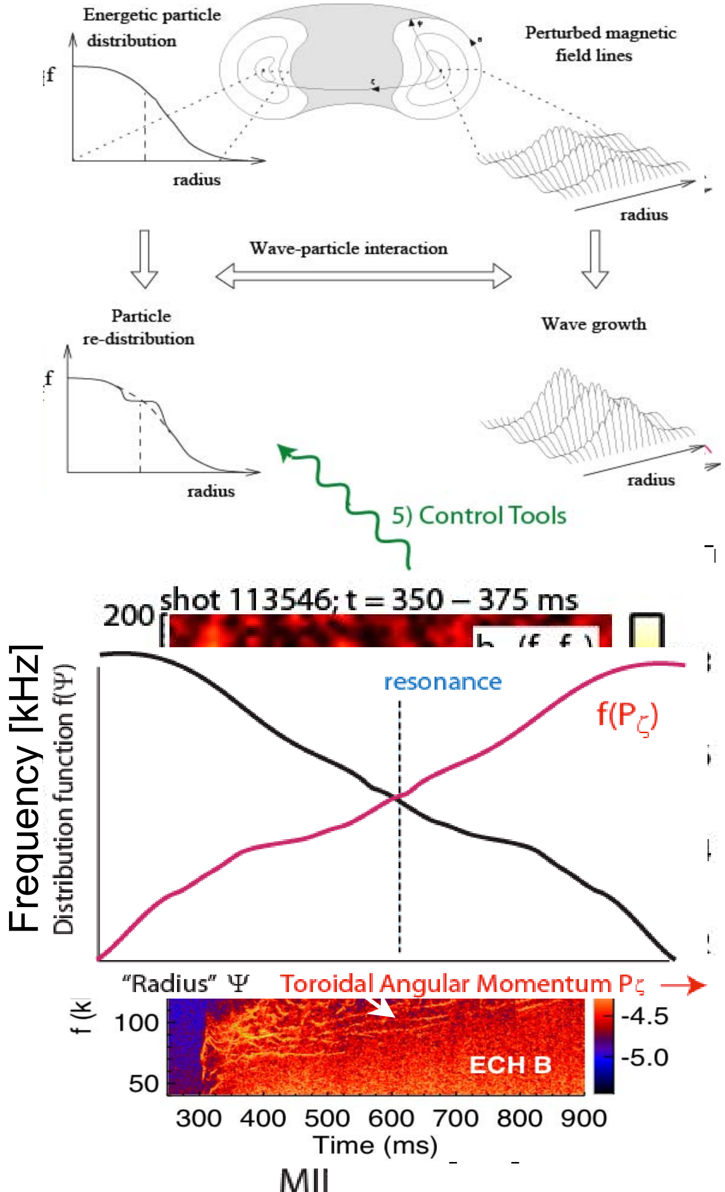
Similar discharges with differing levels of AE activity during beam injection into DIII-D



Wong, Nucl. Fusion 45 (2005) 30.

- Three discharges with different levels of mode activity
- Fast-ion redistribution broadens current profile
- Optimal redistribution triggers an internal transport barrier → much better confinement
- How can we exploit AEs in a reactor?

Conclusions



- •Periodic variations of the index of refraction cause frequency gaps
- •Gap modes exist at extrema of Alfvén continuum
- •Use constants of motion to describe EP orbits
- •Wave-particle resonance occurs when: $\omega - n\omega_\zeta + (m+l)\omega_\theta = 0$
- •Instability driven by EP spatial gradient
- •EPMs are beam modes (not normal modes of background plasma)
- •Berk-Breizman analogy to bump-on-tail problem often describes nonlinear evolution
- •Fast-ion transport not quantitatively understood
- •Use thermal transport techniques to understand nonlinear dynamics
- •Develop tools to control Alfvén instabilities or even improve performance

Acknowledgments* & additional resources

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Brower (UCLA), Crocker (UCLA), Darrow (PPPL), Fasoli (CRPP), Fredrickson (PPPL), García-Muñoz (IPP), Kramer (PPPL), Mauel (Columbia), Nazikian (PPPL), Pinches (UKAEA), Sharapov (UKAEA), Shinohara (JAEA), Snipes (MIT), Spong (ORNL), Toi (NIFS), VanZeeland (GA), Vlad (Frascati), Weller (IPP), White (PPPL), Vann (York), Vincena (UCLA), Zhang (UCI), Zweben (PPPL)

Special thanks for teaching me theory:

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Clear explanation of basic theory: First chapters of Pinches' Ph.D. thesis, http://www.rzg.mpg.de/~sip/thesis/node1.html

Experimental review through 1999 (especially TFTR results): King-Lap Wong, PPCF 41 (1999) R1.

Experimental review of fast ions in tokamaks (AE material dated): Heidbrink & Sadler, NF 34 (1994) 535.

Lengthy theoretical review paper: Vlad, Zonca, and Briguglio, http://fusfis.frascati.enea.it/~Vlad/Papers/review_RNC_2.pdf

Differences between burning plasmas & current experiments: Heidbrink, PoP 9 (2002) 2113

ITER review: Fasoli et al., NF 47 (2007) S264

Recent theoretical review: Chen & Zonca, NF 47 (2007) S727